Inhaled medications are used to treat a variety of respiratory conditions, including chronic bronchitis, asthma, and emphysema. These drug particles are dispersed into the patient’s airways and ultimately absorbed into his/her lungs. However, this treatment is only effective if the particles are able to reach the target site before being eliminated by mucociliary clearance, a process in which the cilia lining the air passages sweep mucus and entrapped particles away from the lungs [1].

The airways are a multi-layer structure comprised of the epithelial surface, the periciliary liquid (PCL), the mucus layer, and air. The cilia, which are immersed in the PCL, exhibit periodic motion. During the effective stroke, the cilia tips penetrate the overlying mucus layer, propelling the mucus and any entrapped particles away from the lungs. The cilia then bend closer to the epithelium and rotate back to their original position; this is the recovery stroke. The work performed during the effective stroke is much greater than the work done during the recovery stroke, thereby posing a challenge for effective transport of drug particles to the lungs [2].

For each cilium and at each time step, we substitute prescribed velocities into equation (3) and solve for the forces exerted by the cilium. The method of images allows us to satisfy the given boundary condition without having to build the wall out of point forces, which could be computationally unwieldy. The result is a matrix-vector equation of the form

\[ \mathbf{u} = \mathbf{M} \mathbf{f} \]

where \( \mathbf{u} \) represents the velocities at the fluid evaluation points and \( \mathbf{f} \) represents the forces exerted by the cilium. \( \mathbf{M} \) is a matrix whose entries depend on the evaluation points, the regularization parameter, and the surface discretization of the cilium [3-5].

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Finally, we substitute these velocities into a discrete-time model that describes the motion of a drug particle subject to advection. The update function we use is

\[ \mathbf{x}_{t+\Delta t} = \mathbf{x}_t + \mathbf{u}(\mathbf{x}_t) \Delta t \]

where \( \mathbf{x}_t \) is the position of the particle at time \( t \).