

# Duke University Math News

May 10, 2005

## Events

### Math Department Party

A large group of undergraduate and graduate students mixed with the faculty at the annual mathematics department party on April 28. On the day after classes ended, members of the Duke mathematical community packed the department lounge to enjoy sandwiches and conversation in an informal setting. This party is the traditional occasion for the faculty to honor the graduating students, contest participants and research students for their hard work and accomplishments. Many math majors received the 2005 Duke Math shirt, some received certificates and a few took home generous cash prizes. A good time was had by all.

### Graduation Luncheon

At noon, immediately after Graduation Exercises on Sunday May 15, senior math majors and their families will meet in the LSRC dining room. In a short ceremony after lunch, those whose first major is mathematics will receive their diplomas.

### Undergrad Math Colloquia

#### Dusa McDuff

On January 31, Dusa McDuff, a mathematics professor at SUNY Stony Brook, gave a colloquium aimed at the level of math majors entitled "4-dimensional polytopes and Symplectic Topology". After this talk, several math majors took her out to a leisurely dinner.

Dusa McDuff earned her Ph.D. in 1971 at Cambridge University. She then studied with Israel M. Gel'fand in Moscow, who taught her to view math as a kind of poetry and whose work greatly influenced her own.

At the State University of New York at Stony Brook, where McDuff has been Professor of

Mathematics since 1978, she has been a major participant in Calculus reform, and has been very active in the Women in Science and Engineering (WISE) program. This program offers academic, financial and social support to female undergraduate students in those fields.

#### Jordan Ellenberg

In a March 1 talk scheduled by DUMU, Jordan Ellenberg of Princeton University and the University of Wisconsin gave an enjoyable talk relating the card game *Set* to questions in combinatorial geometry. A dozen DUMU students enjoyed chatting with him at dinner after his talk.

*Set* is a simple but addictive card game played with a special 81-card deck. A standard "folklore question" among players of this game is: what is the largest number of cards that can be on the table which do not allow a legal play? He explained how this question, which seems to be about cards, is actually about geometry over a finite field. He presented several results and ended with a number of open mathematical problems arising from this study.

Jordan Ellenberg received his bachelor's and doctorate at Harvard University. He won two gold medals in the International Mathematics Olympiad and was twice a Putnam Fellow. More recently, he has received a Sloan Fellowship and an NSF-Career grant.

His research concerns arithmetic algebraic geometry. He writes an occasional column on mathematics for *Slate* magazine and has published the novel "The Grasshopper King."

### High School Math Meet

On Saturday, October 30, the Gross Chemistry Building was the site of the annual Duke University High School Mathematics Meet. At double the attendance of five years ago, 164 students from schools as far as 300 miles away came to compete for the prizes that were given to the top individuals and schools.

The top two individual winners were Thomas Mildorf and Menyong Lee, both of Thomas Jefferson High School in Alexandria, VA. Their high school finished first in the team competition as well. Third place individual prize went to Arnav Tripathy of East Chapel Hill High School with fourth place to Jeff Tang of NCSSM and fifth place to John Pardon of Durham Academy.

The meet is an annual event sponsored by the Duke Math Union (DUMU), currently presided over by Duke senior Oaz Nir. Many thanks go to him and meet organizers Paul Wrayno, Michelle Hu, Nikifor Bliznashki, Keigo Kawaji, and Jackie Ou for their outstanding contribution to the Duke community.

#### **Duke Summer Workshop on Undergraduate Research in Mathematics**

The summer workshop, which will be held May 18-27, 2005, is intended for rising seniors who are interested in graduate study in applied mathematics. The topic for this year's workshop is "Mathematical models of granular flows." The models will describe the dynamics of granular material (e.g., sand, powders, grains, rocks, soil, etc.) in terms of algebraic or differential equations for the mechanics of colliding particles. Over the course of the workshop, the organizers will develop models involving ordinary differential equations for the motion, analyze models to understand their mathematical and physical implications, and conduct numerical and physical experiments to validate the models. The students will work on small research projects related to the topic, and will present their results near the end of the program.

## **Undergraduate News**

### **Putnam Competition Success**

For the fifth time in six years, a team of three Duke students placed third in the annual William Lowell Putnam mathematical competition. In 2000 the Duke team came in first. MIT won this year's competition for the second year in a row. Princeton, coached by Melanie Wood T'03,

a Princeton graduate student, finished second. In 2003, Ms. Wood became the first U.S. woman and only the second woman in the world to be named a Lowell Putnam fellow after finishing in the top five in the competition.

On Saturday December 4, 2004, a record 3733 participants from 515 colleges and universities in the United States and Canada spent six hours working on 12 challenging math problems. Less than half of the participants were credited with even one point out of 120.

For completing 7 of the 12 problems, Niki-for Bliznashki '07 ranked 17th and Oaz Nir '05 and Lingren Zhang '08 were named honorable mention for finishing among the top two percent. Freshmen Kshipra Bhawalkar and Tirasan Khandhawit were among the top 100 and sophomores Brandon Levin and James Zou ranked among the next 100. Morgan Brown '07, Abhijit Mehta '06 and Jason Shapiro '06 were also cited for strong showing. Of the 22 Duke students who took the competition, 17 ranked among the top third of all participants.

The Math Department will receive \$15,000 for this third place finish. A Duke team has finished first, second or third 10 times since 1990 and has finished among the top ten for all but two of those years.

### **Mathematical Contest in Modeling**

In an international competition, held February 4 through 7, 2005, a team of three Duke students won an Outstanding rating in the Mathematical Contest in Modeling (MCM). The contest is held annually, drawing entries from about 750 colleges, universities, and high schools from around the world. This was one of two teams invited by the Mathematics Association of America present their results at the summer meetings of the MAA in Albuquerque NM this August.

Juniors Adam Chandler, Pradeep Baliga, and Matthew Mian worked all weekend to construct mathematical models of traffic flow on a toll road, and to determine the optimal number of lanes for a toll plaza. In their 50 page paper, "The Booth Tolls for Thee," they describe and analyze sophisticated simulations based on queuing theory and cellular automata. After

subjecting their simulations to rush hour conditions on I-95, they concluded that the number of booths should be approximately 1.65 times the number of travel lanes plus 0.9, so as to minimize the time cost to impatient drivers and the expense of operating the booths.

Four other teams from Duke competed in the contest. Three of those teams earned the second highest rating of Meritorious in the MCM: Brandon Levin '07, Matthew Fischer '06, and Nikifor Bliznashki '07; Abhijit Mehta '06, Benjamin Mickle '06, and Ibraheem Mohammed '07; James Zou '07, Oaz Nir '05, and Rahul Satija '06. The remaining team competed in the Interdisciplinary Contest in Modeling (ICM), a new division in the contest. The team of Vyacheslav Kungurtsev '07, Qinzhen Tian '06, and Aaron Wise '08 was Duke's first to compete in the ICM, and they earned a rating of Honorable Mention.

Duke has a long record of success in the MCM, which requires teams of three students to solve an open-ended problem in applied mathematics. Teams have about four days to research the problem, come up with a solution, and write a paper. Duke teams earned Outstanding ratings in 1998, 1999, 2000, 2001, and 2002, and Meritorious ratings in all other years since 1993.

### Goldwater Scholarships

Physics and math major, Peter Blair, and math and chemistry major, Adam Chandler, have been named 2005 B.M. Goldwater Scholars. The \$7500 award is presented annually to about 300 sophomores and juniors for outstanding achievement in mathematics, science and engineering. Since this program began in 1989, 58 Duke students including 25 math majors have been granted this prestigious honor.

Blair, a Mellon Mays Scholar from Nassau, Bahamas, is working with Professor Arlie Peters studying how light bends as it travels around black holes. He is president of the Society of Physics Students at Duke, mentors in the AGAPE Corner Boys Home and sings in a gospel choir.

Chandler, a PRUV Fellow from Burlington, NC, is working with Dr. Garrett Mitchener on modelling language changes. In the Mathematics Contests in Modelling, his teams were designated as Meritorious in 2004 and Outstanding in

2005. He plays cello in the Duke symphony orchestra and has been a Duke tour guide.

For more information, see [www.act.org/goldwater/](http://www.act.org/goldwater/)

### Karl Menger Award

Nikifor Bliznashki '07, Oaz Nir '05 and Lingren Zhang '08 have won the 2005 Menger Award for their excellent ranking in the 2004 the Putnam competition. Each student gets \$250 in addition to the \$600 for their third place finish. Bliznashki and Nir received this award in 2004 as well. As top Putnam scorer in the Southeast region, Bliznashki has been awarded \$100 from the Southeast Section of the Mathematical Association of America. Zhang, a first year student from Shanghai, will receive an additional \$250 for winning the Virginia Tech Math Contest in October 2004 and Nir will collect \$200 for finishing third.

Karl Menger (1902-1985), a member of the Vienna Circle of Logical Positivism that flourished around 1930, was a renowned mathematician who made significant contributions to areas of logic, philosophy and economics as well as mathematics. Among his students in Vienna was Kurt Gödel. After leaving Europe in 1937, Menger taught at Notre Dame and then the Illinois Institute of Technology until he retired in 1971. For more information see [www.math.duke.edu/news/awards/menger/index.html](http://www.math.duke.edu/news/awards/menger/index.html)

### Julia Dale Prize

The 2005 Julia Dale Award for excellence in mathematics is shared by Oaz Nir and Mayank H. Varia.

Nir is an A. B. Duke Scholar, PRUV Fellow, Putnam team member for three years and two-term president of DUMU. His senior thesis, under the direction of John Harer, is entitled "Mechanical arms and algebraic topology." He will be attending graduate school in applied mathematics at MIT.

Varia is a NC Math Scholar who has excelled in his course work at Duke. His senior thesis, under the direction of Jonathan Hanke, is entitled "Explicit computation of the L function of a Kummer surface". He will be attending graduate school in pure mathematics at MIT.

Julia Dale (b. 1893) was a professor of mathematics at Duke from 1930 until her untimely death in 1936. Her friends and relatives established the Julia Dale Memorial Fund in her honor. For more information, see [www.math.duke.edu/news/awards/dale/](http://www.math.duke.edu/news/awards/dale/)

### PRUV Research Fellows

The PRUV Research program, now in its sixth year, supports undergraduates for six weeks of intensive math research under the direction of a Duke professor. Each of these students is expected to write a senior thesis worthy of graduation with distinction. For more information, see [www.math.duke.edu/vigre/pruv/index.html](http://www.math.duke.edu/vigre/pruv/index.html). The following students have been selected as PRUV Fellows for the summer of 2005.

- Nikifor Bliznashki [with W. Pardon]
- Vyacheslav Kungurtsev [with M. Huber]
- Brandon Levin [with L. Saper]
- Peter Merx [with H. Bray]
- Ibraheem Mohammed [with A. Petters]
- Qinzhen Tian [with T. Witelski]
- Gabriel Williams [with A. Petters]
- Yee Lok Wong [with J. Mattingly]

### Undergraduate Courses for Fall 2005

This fall, Math 196S (Seminar in Mathematical Modeling) will focus on applications of mathematics to solving models of physical fluid dynamics.

This course provides an introduction to the mathematical modeling of modern problems in fluid dynamics. Beginning with the derivation of the fundamental equations of fluid motion, we will construct models for waves and free-surface flows, lubrication theory and the shallow water equations. Techniques for solving these problems analytically and numerically will be developed with an emphasis on gaining an insight into the underlying physics. We will cover topics including hyperbolic and parabolic partial differential equations, nonlinear ordinary differential

equations, boundary layers, dimensional analysis, and stability theory.

This course will be co-taught by Professor Tom Witelski and Dr. Rachel Levy. The prerequisites are Math 103, physics (mechanics) and differential equations (Math 107/108 or 111 or 131).

For more information, see ACES or [www.math.duke.edu/witelski/196/](http://www.math.duke.edu/witelski/196/). This course is partially supported by research grants from the National Science Foundation and will offer continuing opportunities for summer undergraduate research projects in mathematical analysis and experimental physics.

### ACM World Finals

A team of three Duke students have distinguished themselves in the 2005 ACM World Finals. Garrett Casto '05, Ben Mickle '06 and Matt Edwards '08 traveled to Shanghai in early April to compete with 77 other teams culled in preliminary rounds from over 1500 teams from 71 countries. Four of 22 teams from the United States finished among the top 40 in what is claimed to be the oldest, largest and most prestigious programming contest in the world. Duke tied with MIT and Cal Tech and among US universities, only the University of Illinois ranked strictly higher. Casto will graduate with a major in BME and CS. Mickle is a CS and Math major who who teamed with other math majors earlier this year to gain a Meritorious ranking in the Math Contest in Modelling. Freshman Edwards also expects to major in CS and Math. For more information, see <http://icpc.baylor.edu/icpc/>.

## Graduate Program News

### Graduating Ph.D Students

Robert Buckingham has accepted an Assistant Professorship at the University of Michigan at Ann Arbor. His thesis "Long-Time Asymptotics of the Nonlinear Schrödinger Equation Shock Problem" was written under the direction of Stephanos Venakides.

John Cain has accepted an Assistant Professorship at Virginia Commonwealth University. His

thesis "Issues in the One-Dimensional Dynamics of a Paced Cardiac Fiber" was written under the direction of David Schaeffer.

Carina Curto has accepted a Post-Doc Position at Rutgers University. Her thesis "Matrix Model Superpotentials and Calabi-Yau Spaces: an ADE Classification" was written under the direction of David Morrison.

Andrew Feist wrote his thesis on "Two Problems in Delay Differential Equations" under the direction of Thomas Witelski.

Daniel Fox has accepted a Visiting Assistant Professorship at the University of California at Irvine. His thesis "Second Order Families of Co-associative 4-Folds" was written under the direction of Robert Bryant.

Guoqiang Yang wrote his thesis "Quantitative Models for Dorsal closure in Drosophila Embryos" under the direction of Stephanos Venakides.

Dan Yasaki has accepted a Visiting Assistant Professorship at the University of Massachusetts. His thesis "On the Existence of Spines for Q-rank 1 Groups" was written under the direction of Les Saper.

Wenjun Ying will be a Research Associate at Duke University. His thesis "A Multilevel Adaptive Approach for Computational Cardiology" was written under the direction of John Trangenstein.

## Faculty News

### Richard Scoville

Richard Scoville, associate professor emeritus, died on November 26, 2004, of lung cancer. He had lived for 25,488 days.

Scoville was the first of his family to attend college, receiving a full-tuition scholarship to Yale University. There he earned his bachelors, masters and Ph.D. degrees, all in mathematics.

Scoville joined the Duke Mathematics Department in 1961 and continued as a faculty member until he retired in 1998. He served as director of undergraduate studies from 1992 until 1995. In that role, he edited the first issues of this *Duke*

*Math News*. He was the author or co-author of 38 professional articles in the fields of combinatorics and ergodic theory, many written jointly with the late professor Leonard Carlitz.

After retiring from Duke, he volunteered his time to help elementary school students with reading and to instruct GED classes. He was interested in, talented in, or fond of: Japanese and Spanish language and culture, the game of Go, classical music, stamp collecting, photography, sports cars, gardening, astronomy, entomology and etymology, computers and robotics, Bonsai, cheesecake, butterscotch pudding and maple sugar candy.

Surviving Scoville are his son Steven and daughter Satsuki (from his marriage with Takae Scoville), their spouses, three grandchildren, a sister, nieces and nephews, and his long-time companion, Alice Davidson. Friends, neighbors, relatives and former colleagues gathered to remember him at a memorial service in January.

### Promotions

Leslie Saper will be promoted to professor and Thomas Witelski to associate professor effective this fall. Saper's research involves locally symmetric varieties, automorphic forms, L2-cohomology and intersection cohomology, and geometrical analysis of singularities. Witelski's research centers on use of nonlinear differential equations to describe the complicated dynamics of fluids in various physical situations.

## Problem Corner

### Solutions from Last Issue

- Problem 1.** Prove that if  $7|a^2 + b^2$ , then  $7|a$  and  $7|b$ .

*Solution* One can easily check that  $a^2$  gives remainders  $\{0, 1, 2, 4\}$  when divided by 7, therefore  $a^2 + b^2$  can be divisible by 7 if and only if  $a^2$  and  $b^2$  give remainders 0. Since 7 is prime, it follows that  $7|a$  and  $7|b$ .

- Problem 2.** Let  $a, b, m$  and  $n$  be natural numbers with  $a$  and  $b$  relatively prime. Show that  $(a^n - b^n)|(a^m - b^m)$  if and only if  $n|m$ .

*Solution* First, let us suppose that  $n|m$  and  $m = nk$ . Then  $a^m - b^m = (a^n)^k - (b^n)^k = (a^n - b^n)((a^n)^{k-1} + \dots + (b^n)^{k-1})$  which is clearly divisible by  $a^n - b^n$ . Conversely, suppose that  $(a^n - b^n)|(a^m - b^m)$ . Then  $(a^n - b^n)|(a^m - b^m) - a^{m-n}(a^n - b^n) = (a^{m-n}b^n - b^m) = b^n(a^{m-n} - b^{m-n})$ . Since  $(a, b) = 1$  it follows that  $(a^n - b^n)|(a^{m-n} - b^{m-n})$ . If we do this  $q$  times, we obtain that  $(a^n - b^n)|(a^{m-nq} - b^{m-nq})$ . From the division theorem  $m = nq + s, 0 \leq s < n$  for some  $q, s$  and therefore  $(a^n - b^n)|(a^s - b^s)$ . But  $\frac{a^n - b^n}{a - b} = (a^{n-1} + a^{n-2}b + \dots + b^{n-1}) > (a^{s-1} + a^{s-2}b + \dots + b^{s-1}) = \frac{a^s - b^s}{a - b}$  and therefore  $a^s - b^s = 0$ . Since  $(a, b) = 1$  it follows that  $s = 0$  and therefore  $n|m$ .

- Problem 3.** Show that the polynomial

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

is irreducible in  $\mathbb{Q}[x]$  for every natural number  $n$ .

*Solution* From Gauss's Lemma, it is sufficient to show that

$$n!f(x) = x^n + nx^{n-1} + n(n-1)x^{n-2} + \dots + n!$$

is irreducible over  $\mathbb{Z}[x]$ . From Chebyshev's theorem, there is a prime number  $p$  such that  $m < p \leq 2m$  for every natural number  $m$ . If  $n$  is even,  $n = 2k$  for some natural  $k$ . Then between  $k$  and  $2k$  there is a prime number  $q$  and  $q < n < 2q$ , therefore  $q|n$  but  $q^2 \nmid n!$ . If  $n$  is odd,  $n = 2k + 1$  and take again a prime number  $q$  between  $k$  and  $2k$ . Then  $q \leq n - 1 < 2q$ . Since  $n - 1$  is even and  $2q$  is even, it follows that  $n - 1 < 2q - 1$  and therefore  $q < n < 2q$ . Again, we can conclude that  $q|n$  and  $q^2 \nmid n!$ . Now, if we apply Eisenstein's criteria to  $n!f(x)$  for  $q$ , we obtain that  $n!f(x)$  is irreducible in  $\mathbb{Z}[x]$  and therefore  $f(x)$  is irreducible in  $\mathbb{Q}[x]$ .

### New Problems

- Problem 1** Find all  $n$  for which  $\underbrace{10101 \dots 10101}_{2n-1 \text{ digits}}$  is prime.
- Problem 2** Let  $f(x)$  be a polynomial with integer coefficients, and let  $\alpha_1, \alpha_2, \dots, \alpha_n$  be its complex roots. Prove that if  $f(a)$  is prime for some integer  $a$  with  $|a| > \max|\alpha_i| + 1$ , then  $f(x)$  is irreducible in  $\mathbb{Z}[x]$ .
- Problem 3** Prove that

$$\sum_{k=0}^m \binom{m}{k} \binom{n+k}{m} = \sum_{k=0}^m \binom{m}{k} \binom{n}{k} 2^k.$$

Submit solutions or suggestions for new problems to Problem Editor Nikifor Bliznashki [nikifor.bliznashki@duke.edu](mailto:nikifor.bliznashki@duke.edu)

## Math Degree Candidates, Academic Year 2004-2005

### First Majors

Jennifer Lynn Beall  
 William E Bigner  
 Andrew Philip Card  
 Stephen Harrison Coit  
 Justin Kyle Davis  
 Michael Faber  
 Pauline Hong  
 Thomas William Horn  
 Paul George Jameson  
 Emily Efrosini Khoury  
 David Lawrence Mermin  
 Colin B Middleton  
 Benjamin Andrew Morgan  
 Ryan Chase Morton-Wurst  
 Oaz Nir  
 Emily Marie Page  
 Lauren Ashleigh Price  
 Jason Aaron Schanker  
 Ashley Allison Siebert  
 Tadena A Simpson  
 Ann-Drea Marie Small  
 Lauren Marie Smith

### Second Majors

Christ T Balich  
 Krupal Pravin Bhatt  
 Dennis Sean Casey  
 Nicolette Chao  
 Quan Cheng  
 Michael N Economo  
 Michelle Hu  
 Pooja Nemichand Jain  
 Ethan Thomas Neil  
 Merrill J Roller  
 Benjamin David Seidman  
 Mayank Harshad Varia

### Third Major

Matthew M Engelhard

### Minors

Juan Pablo Bermudez  
 Stephen Trent Corbin  
 Christopher Lee Cox  
 Danielle M Davidian  
 John Russel Denton  
 Grant Stephen Gilliam  
 Claire Louise Grandadam  
 Rachel Merritt Heath  
 Megan Clare Leftwich  
 Supanika Leurcharusmee  
 Lauren A Lind  
 David Ross Martin  
 Emily Jayne McDowell  
 Michele E Pugh  
 Jonathan Howard Rick  
 Denise Gayle Rotatori  
 Shayla Charelle Sanford  
 Isaac E Specter  
 Colleen Michelle Torke  
 Charles Pai Wang  
 Jae-Hoon Yoon

### Master of Arts

Rann Bar-on  
 Sergei Belov  
 Mihaela Froehlich  
 Conrad Hengesbach  
 Jenny Law  
 Abraham Smith

### Ph.D

Robert Buckingham  
 John Cain  
 Carina Curto  
 Timothy Deering  
 Andrew Feist  
 Gregory Firestone  
 Daniel Fox  
 Guoqiang Yang  
 Dan Yasaki  
 Wenjin Ying

## Duke Math News

The *Duke Math News* is published several times a year and is distributed to those in the Duke mathematics community by campus mail. For previous editions and other news, see [www.math.duke.edu/news/](http://www.math.duke.edu/news/). We welcome items of interest for our next issue. Send them to [jones@math.duke.edu](mailto:jones@math.duke.edu) or [dkrain@math.duke.edu](mailto:dkrain@math.duke.edu)

To read about other news, honors and events concerning mathematics at Duke, visit [www.math.duke.edu/news/](http://www.math.duke.edu/news/). The on-line calendar at [www.math.duke.edu/mcal](http://www.math.duke.edu/mcal) lists both regular and special seminars and colloquia for the upcoming weeks. The department maintains video archives of talks, lecture series and special conferences at Duke, many of which are available, on-line. See [www.math.duke.edu/computing/broadcast.html](http://www.math.duke.edu/computing/broadcast.html) for more information.

—*David Kraines, DMN Faculty Sponsor*

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