

# Duke University Math News

May 5, 2004

## Events

### 3rd Duke Mathematical Journal Conference

The 3rd Duke Mathematical Journal conference was held at Duke University from April 23 through April 25, 2004. The conference brought young mathematicians together for a weekend of high quality mathematics, with talks given by eight of their peers. The speakers at this conference were:

- Denis Auroux (MIT), Homological mirror symmetry for Fano varieties
- Manjul Bhargava (Clay Institute & Princeton), Finiteness theorems for quadratic forms
- Alexei Borodin (Clay Institute & CalTech), Markov chains on partitions, Plancherel measures, and polynuclear growth processes
- Tobias Colding (Courant Intitute, NYU), The Calabi-Yau conjectures for embedded surfaces
- Kiran Kedlaya (MIT), Some p-adic differential equations in arithmetic geometry
- Ezra Miller (Minnesota), Combinatorial positivity by geometric degeneration
- Mircea Mustata (Clay Institute), On some invariants of singularities
- Gigliola Staffilani (MIT), Global well-posedness and scattering for the energy-critical nonlinear Schrödinger equation in  $\mathbb{R}^3$

### 1st Triangle Discrete Math and Optimization Day

The 1st Triangle Discrete Math and Optimization Day was held on March 22 at the Fuqua School of Business. The aim of this conference was to facilitate communication between researchers, graduate and undergraduate students from around the Research Triangle who have interests ranging among Algebraic and Enumerative Combinatorics, Discrete Geometry, Graph Theory, Coding and Design Theory, Combinatorial Aspects of Computational Algebra and Geometry, Combinatorial Optimization and Integer Programming, Probabilistic Combinatorics, and Combinatorics in Mathematical Physics.

This inaugural meeting was organized by Christian Haase, Gabor Pataki of UNC and Aleksandar Pekec of the Fuqua School. The main speakers were Louis Billera from Cornell, Martin Grötschel from Berlin, and Carla Savage from NC State.

Details at [www.or.unc.edu/pataki/optday/](http://www.or.unc.edu/pataki/optday/)

### Graduation Luncheon

Graduating students who major in mathematics and their families are cordially invited to a luncheon in the Levine Science Research Center dining room after the Graduation Exercises on Sunday May 9. In a brief ceremony at the end of the luncheon, Director of Undergraduate Studies, Richard Hodel, will present the diplomas to those with math as their first major.

## Undergraduate News

### Putnam Competition Success

For the fourth time in five years, a team three Duke students placed third place in the annual William Lowell Putnam mathematical competition. On Saturday December 6, 2003, a record

3615 participants from 479 colleges and universities in the United States and Canada spent 6 hours working on 12 challenging math problems. Only 1/4 of the participants nationwide managed to solve even one problem completely. Of the 25 Duke students who took the competition, 17 ranked among the top quarter and 9 placed in the top 11%.

David Arthur '04 ranked 9th in the nation for completing 8 problems while Nikifor Bliznashki '07 and Oaz Nir '05 were named Honorable Mention for finishing among the top 1%. The team of Arthur, Bliznashki and Nir finished behind MIT and Harvard to capture the \$15,000 third place prize. Duke teams have won the competition three times, finished second twice and third four times since 1990.

Sample problem: Find the minimum value of  $|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$

Ans:  $2\sqrt{2} - 1$

### Mathematical Contest in Modeling

A record four Duke teams successfully competed in the 2004 Mathematical Contest in Modeling. On Thursday, February 5, nearly 600 teams of three undergraduates from around the world began their research on one of two problems. Four days later, the teams submitted their detailed mathematical analysis of the problem.

Two Duke teams were declared Meritorious, ranking among the top 10% of submissions. Sophomores Pradeep Baliga, Adam Chandler and Matthew Mian developed a "quick pass" system that could be used by amusement parks to reduce waiting time for rides. In their 47 page paper, they included a statistical analysis and computer simulation of their system. Abhijit Mehta '06, Oaz Nir '05 and James Zou '07 developed an algorithm for identification of fingerprints that would require fewer measurements than are currently required and for which the probability of a mismatch is roughly the same as for DNA evidence. The teams of Nikifor Bliznashki '07, Matthew Fischer '06, and Brandon Levin '07, and of Morgan Brown '07, Will Horn '05, and Ethan Neil '05 earned Honorable Mention for their efforts.

Duke alumnus, Dr. W. Garrett Mitchener '99, was the faculty advisor of these teams. As a student, Mitchener earned Meritorious status twice and led two teams that were named Outstanding Winners, the highest honor of this competition. The competition is sponsored by the Consortium for Mathematics and its Applications (COMAP) and receives major funding from the National Security Agency. For more information, see <http://www.comap.com/undergraduate/contests>.

### Goldwater Scholarships

Math and Biology major Jacqueline Ou '06 was awarded the B. M. Goldwater Scholarship this year. The \$7500 scholarship is presented annually for outstanding achievement in mathematics, science and engineering. Jackie has conducted bioinformatics research at the Cold Spring Harbor Biological Laboratories and in the molecular genetics laboratory at Duke. Since the program began in 1989, 52 Duke students including 23 math majors have been granted this prestigious honor.

### Karl Menger Award

The Duke Putnam team of David Arthur '04, Nikifor Bliznashki '07 and Oaz Nir '05 earned the Menger award for their placement among the top 1% of the contestants in that competition. Each student received \$250 in addition to a \$600 check from the Putnam Foundation for the team's third place finish. This award is granted in honor of Karl Menger, an Austrian born mathematician who made significant contributions to areas of logic, philosophy and economics as well as to pure mathematics.

### Morgan Prize

Melanie Wood '03 was named recipient of the 2003 Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student. Wood received a \$1000 cash prize and a certificate at the Joint Mathematics Meetings in Phoenix last January. The selection committee cited Melanie's honors thesis at Duke on combinatorial structures associated with algebraic curves defined over number fields, done under the supervision of Richard Hain, and her work on algebraic number theory, done at

the Duluth REU in 2000. Her work on algebraic curves yields important insights into the action of the absolute Galois group on fundamental groups. The Prize committee called her work deep and original. Wood will enter the Princeton graduate program in math this fall after finishing her studies at Cambridge University. (adapted from [www.maa.org/news/morganprize04.html](http://www.maa.org/news/morganprize04.html))

### Julia Dale Prize

Each year, the Julia Dale prize is awarded to one or more students for excellence in mathematics. A cash prize in honor of former Duke professor Julia Dale was given to graduating seniors David Arthur, Ryan Letchworth, David Marks and Lindsay Piechnik. Each of these students will graduate with distinction for their mathematical research papers. Stanford graduate school will be the new home for Arthur (computer science) and Marks (physics). Letchworth will join the international investment firm of Dresdner, Kleinwort and Wasserstein in New York as an investment banking analyst and Piechnik will study mathematics at Columbia University.

### PRUV Research Fellows

Nine math majors will graduate with distinction this spring, the largest number ever at Duke. Each of these students wrote an undergraduate thesis in pure or applied mathematics under the direction of a Duke professor while supported by a grant from the National Science Foundation through the PRUV Fellow program.

- David Arthur *On the higher Hasse-Witt matrices and related invariants* [with W. Pardon] \*
- Suzy Borgschulte *A mathematical approach to the panting of dogs*[with M. Reed]
- Lauren M. Childs *Scaling population dynamics from the macroscopic to the microscopic* [with T. Kepler]
- Ryan Letchworth *Wavelet methods for numerical solutions of differential equations* [with S. Roudenko] \*\*

- David Marks *Coadjoint orbits and geometric quantization*[with M.R. Plessner]
- Lori Peacock *Distributions of the small eigenvalues of Wishart matrices* [with B. Rider]
- Lindsay C. Piechnik *Smooth reflexive 4-polytopes have quadratic triangulations*[with C. Haase] \*
- Matthew Toups *A solution to the D0-D4 system of equations*[with M. Stern]
- Jenna VanLiere *Mathematically modelling the growth and diversification of T-cell populations*[with T. Kepler]

\* Graduation with High Distinction

\*\* Graduation with Highest Distinction

## Graduate Program News

### Graduating Ph.D Students

Andrew Barnes wrote his thesis "Electromagnetic Scattering from three dimensional periodic structures" under the direction of Stephanos Venakides.

Christian Benes accepted a 3-year assistant professorship at Tufts University. His thesis "On Some Problems Concerning Random Walks", was written under the direction Greg Lawler.

John Greer will be at the Courant Institute on an NSF fellowship starting this September. His thesis "Fourth order diffusions for image processing" was written under the direction of Andrea Bertozzi.

Chris Hale wrote his thesis "On Time-Domain Calculations of Leaky Modes in Photonic Crystals" under the direction of Stephanos Venakides.

Kevin Kessler wrote his thesis "Analysis of feedback-mediated dynamics in two coupled nephrons" under the direction of Harold Layton.

Michael J Kozdron has accepted a tenure-track assistant professorship with the Department of Mathematics and Statistics at the University of Regina in Regina, Saskatchewan, to commence

on July 1, 2004. His thesis "Simple Random Walk Excursion Measure in the Plane" was written under the direction Greg Lawler.

## Faculty News

### Joe Kitchen

Associate professor Joseph Kitchen retires this year after 40 years of service to the mathematics department. He arrived at Duke after receiving his BA and Ph.D. from Harvard. Kitchen has won University teaching awards and has long been considered one of the most popular teachers in the math department. His lectures were always efficiently organized and characterized by honesty, precision, and clarity. The same characteristics may be found in his text, *Calculus of One Variable*, published by Addison-Wesley and intended for honors calculus courses. His research papers are in the area of Functional Analysis and include a series of papers on Banach bundles written with his former student David A Robbins.

An accomplished pianist and organist, Kitchen has given solo recitals on both instruments at Duke. For over thirty years he was organist and choirmaster at St. Stephen's Episcopal Church. He also performed piano chamber music in the United States and Europe with many professional musicians including Luca and Arlene DiCecco, founding members of the Ciompi Quartet, and, most notably, with his son, violinist Nicholas Kitchen, and his daughter-in-law, cellist Yeesun Kim. For four years Kitchen was accompanist for the Duke Chorale.

In retirement Kitchen plans to continue his musical activities. Presently he is serving once again as organist at St. Stephen's. Other activities will include travel, reading, cooking, working-out, and gardening. He will travel to Boston from time to time to help with his new grandson Christopher who turned one on April 4.

### Lang Moore

Associate professor Lawrence (Lang) Moore will retire from Duke this spring. After his undergraduate years at NC State University, Moore studied mathematics at California Institute of Technology and received his Ph.D. in 1966. He then began his career at Duke where he has been ever since except for several sabbatical leaves. Moore has published papers in the theory of ordered spaces and, in collaboration with Ward Henson, in the area of application of model theory to functional analysis and has supervised four Ph.D. students while at Duke.

More recently, Moore's major scholarly interests have turned to curriculum development, in particular, the development of web-based learning materials. With associate professor emeritus David Smith, he contributed significantly to the calculus reform movement nationally and at Duke. In 1991, Moore and Smith were awarded the EDUCOM Higher Education Software Award for "Best Mathematics Curriculum Innovation" in recognition of their development of Project CALC.

Currently, Moore is the Executive Editor of the Mathematical Sciences Digital Library (MathDL), an NSF-sponsored project of the Mathematical Association of America. He and Smith will continue to develop an online interactive version of their calculus book and will work on other pedagogical projects under the auspices of the Mathematical Association of America.

Moore continues to enjoy reading at his beach house, attending Duke basketball games, and visiting with his grandchildren.

## Problem Corner

### Solutions from Last Issue

1. *Problem 1.* Assume that the opposite angles of a convex quadrilateral sum to  $180$ . show that the quadrilateral can be inscribed in a circle.

*Solution* In the quadrilateral ABCD, let  $\theta = \angle ABC$  so that  $\angle CDA = \pi - \theta$ . In the circle circumscribed about the triangle ABC,  $\angle ABC$  subtends an arc of measure  $2\theta$ . Since the complementary arc has measure  $2\pi - 2\theta$ , D must lie on the circle.

2. *Problem 2.* Let  $x_1, x_2, x_3, x_4$ , and  $x_5$  be real numbers. Find the smallest natural number  $n$  with the following property: if any  $n$  different sums  $x_p + x_q + x_r$ ,  $1 \leq p < q < r \leq 5$ , are equal to zero, then  $x_1 = x_2 = x_3 = x_4 = x_5 = 0$ .

*Solution* We will show that  $n = 7$ . First, we will give an example with  $n = 6$  when the statement is not true. Let's take  $x_1 = x_2 = 4$ , and  $x_3 = x_4 = x_5 = -2$ . Then any sum of the form  $x_i + x_j + x_k$ , where  $i = 1$  or  $2$ ,  $j$  and  $k = 3, 4$ , or  $5$ , is equal to zero. The number of ways we can choose  $i$  is 2, the number of ways we can choose  $j$  and  $k$  is 3 and therefore we have 6 different zero sums, with not all  $x_i = 0$ . Therefore,  $n \geq 7$ .

Now, let's take  $n = 7$ . We will prove that  $x_i = 0$  for all  $i$  between 1 and 5. Let  $A = \{\{x_p, x_q, x_r\} | 1 \leq p < q < r \leq 5\}$  be the set of all possible 3-sets and  $B \subset A$  the set of triples with zero sums. Since  $n = 7$ , we have that  $B$  has 7 elements. Since we have 7 sums that are equal to zero, and in each one of them are 3 numbers, there are 21 numbers participating in the sums. But we only have 5 numbers and therefore there is a number that participates in at least 5 of the sums. Assume, without loss of generality that this number is  $x_1$ . Let  $C = \{\{x_p, x_q\} | \{x_1, x_p, x_q\} \in B\}$ . We have that  $C$  has at least 5 elements, and there are 4 numbers that can be elements of an element of  $C$ , therefore there is at least one number that participates in two of the elements of  $C$ . Assume, without loss of generality that this element is  $x_2$  and it participates along with  $x_3$  and  $x_4$ .

Case 1:  $\{x_2, x_5\} \in C$ . Then

$x_1 + x_2 + x_3 = x_1 + x_2 + x_4 = x_1 + x_2 + x_5 = 0$  and therefore  $x_3 = x_4 = x_5 = x$ . But  $C$  has at least two elements not involving  $x_2$  and therefore we have that  $x_1 + x + x = 0$ , from

where  $x_1 = -2x$ ,  $x_2 = x$ . Now we recall that  $B$  has 7 elements, and we can choose 2 numbers out of 4 in 6 different ways, therefore  $x_1$  does not participate in at least one of the elements of  $B$ , so we have a sum in the form  $x_p + x_q + x_r = 0$  where  $p, q$  and  $r$  are between 2 and 5, therefore  $3x = 0$ , so  $x = 0$ , from where  $x_1 = x_2 = x_3 = x_4 = x_5 = 0$ .

Case 2:  $\{x_2, x_5\} \notin C$ . Then  $C$  has 3 more elements, none of which involve  $x_2$ , and therefore they are  $\{x_3, x_4\}, \{x_3, x_5\}, \{x_4, x_5\}$ . This implies that

$$\begin{aligned} x_1 + x_3 + x_4 &= 0 \\ x_1 + x_3 + x_5 &= 0 \\ x_1 + x_4 + x_5 &= 0 \end{aligned}$$

from where  $x_3 = x_4 = x_5 = x$ ,  $x_1 = -2x$ , and we have again that  $x_2 = x$ . As in the previous case, there is at least one sum not involving  $x_1$  and therefore  $x = 0$ , so all the numbers should be zero.

Therefore, the answer is  $n = 7$ .

3. *Problem 3.* Prove that the equation  $y^2 = x^5 - 4$  has no solution in natural numbers.

*Solution* From Fermat's little theorem,

$$x^{11} \equiv x \pmod{11}$$

therefore  $11 | x(x^5 - 1)(x^5 + 1)$ , and since 11 is prime, divides at least one of the numbers  $x, x^5 - 1, x^5 + 1$ , so

$$x^5 \equiv 0, \pm 1 \pmod{11}$$

from where follows that the right side of the equation gives remainders 6, 7, or 8 when divided by 11. However, the remainders of  $y^2$  can only be 0, 1, 3, 4, 5, or 9, and therefore the right side cannot be equal to the left side of the equation, so it has no solutions.

## New Problems

1. *Problem 1* We will call an integer number  $n$  "good" if it can be represented in the form  $n = x^2 + y^2$ , where  $x$  and  $y$  are integers. show that if  $a$  and  $b$  are "good", then their product  $ab$  is also a "good" number.
2. *Problem 2* Find all natural numbers  $n$  such that  $n|2^n - 1$  (Balkan Mathematical Olympiad).
3. *Problem 3* Is there a function  $f(x) : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(f(x)) = x^2 - 2004$ ? (Towns' Tournament)

Submit solutions or suggestions for new problems to Problem Editor Nikifor Bliznashki  
 nikifor.bliznashki@duke.edu

## Math Degree Candidates, Academic Year 2003-2004

### First Majors

David Greig Arthur  
 Sean Christopher Behre  
 Laura Ann Bohn  
 Suzanne Elizabeth Borgschulte  
 Anna Ruth Burkhead  
 Lauren Maressa Childs  
 Elena Jane Edelman  
 Evan Ira Ettinger  
 Lauren Talanda Fath  
 Thora Bjorg Helgadottir  
 Genevieve Anne Hildebrand  
 Joseph Patrick Huffsmith  
 Erik Boyd Johnson  
 Matthew Ross Kaufmann  
 Ahmed Mosharraf Khan  
 Christopher Edward Krzeminski  
 David William Marks  
 D'Juan Edward O'Donald  
 Shaun Nicholas Pack  
 Lori Beth Peacock  
 Lindsay Carter Piechnik  
 Emily Ruth Reither

Michael William Robbins  
 Ira Hyman Schachar  
 Taren Kate Stinebrickner-Kauffman  
 Jenna Marie VanLiere  
 Bryan Justin Whitfield

### Second Majors

Ethan Duff Eade  
 Eric Michael Fountain  
 Megan Rose Gessner  
 John Paul Marnell  
 Codin Catalin Pora  
 Adam Frank Siegel  
 Matthew Henry Touns  
 Marc Darren Winniford

### Third Majors

Christopher John Dillenbeck  
 Ryan Lucas Letchworth

### Minors

David Louis Beckman  
 Alexa Megan Berlin  
 Wagish Bhartiya  
 Steven Michael Bischoff  
 Dana Olcott Burwell  
 Abigail Benita Cheung  
 Daryl C. Hare  
 Seth Emerson Lankford  
 Theodore David Dexter Mann  
 Robert Bradley McDowell  
 Dmitri V Mirovitski  
 Jonathan Robert Perlstein  
 Joel Chase Rogers  
 Sezin Topak  
 Jessica Lynne Vodofsky

**Master of Arts**

Michael Gratton  
 John Hyde  
 William LeFew  
 James Michael  
 Joseph Spivey  
 Philip Vetter  
 Feng Xu

**Ph. D**

Andrew Barnes  
 Christian Benes  
 John Greer  
 Christopher Hale  
 Kevin Kessler  
 Michael Kozdron

**Duke Math News**

The *Duke Math News* is published several times a year and is distributed to those in the Duke mathematics community by campus mail. For previous editions and other news, see [www.math.duke.edu/news/](http://www.math.duke.edu/news/). We welcome items of interest for our next issue. Send them to [jones@math.duke.edu](mailto:jones@math.duke.edu) or [dkrain@math.duke.edu](mailto:dkrain@math.duke.edu)

To read about other news, honors and events concerning mathematics at Duke, visit [www.math.duke.edu/news/](http://www.math.duke.edu/news/). The on-line calendar at [www.math.duke.edu/mcal](http://www.math.duke.edu/mcal) lists both regular and special seminars and colloquia for the upcoming weeks. The department maintains video archives of talks, lecture series and special conferences at Duke, many of which are available, on-line. See [www.math.duke.edu/computing/broadcast.html](http://www.math.duke.edu/computing/broadcast.html) for more information.

—*David Kraines, DMN Faculty Sponsor*

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