

# Duke University Math News

September 19, 2000

## Welcome

First, a big welcome to all the newcomers — all of the new postdocs and visitors, all of the new graduate students, and Janet, Georgia and Sunny in the front office. I hope you will all enjoy working and studying in the department.

As many of you know, I became chair a year ago when John Harer, our previous chair, accepted an appointment as Vice Provost for Academic Affairs. This last year has been both challenging and educational for me, with university-wide strategic planning and an external review. I never cease to be amazed at the vitality and diversity of activities in the Department — everything from the high level of teaching, the vigorous research activity, and the great success in raising outside funding to support graduate students, research, and curriculum development. Our undergraduates have continued their successes on national competitions, such as the Putnam and Modeling competitions, while our graduate program is, once again, ranked in the top 25 by US News and WorldReport.

Those returning will see that there have been many changes over the summer. The most obvious will be the major renovations of the administrative space in Room 121 and the creation of seven new offices where the library was previously located. The new offices will help to alleviate our space problem and to accommodate an increased number of graduate students, postdocs, and visitors.

The Mathematics-Physics Library has been moved to the Teer Building next door. It has been combined with the Engineering Library and is now called the *Vesic Engineering Library*. It will have extended opening hours as well as 24/7 card access for graduate students and faculty. Mary Ann Southern and Judith Bailey will work in the new library under Linda Martinez, who heads the Science and Engineering libraries at

Duke.

Less tangible, but very significant, is the new VIGRE (*Vertical Integration of Research and Education*) grant the Department was awarded by the National Science Foundation (NSF). The VIGRE funds are primarily to improve the graduate program through better research training and increased interaction with postdocs. With help from the VIGRE funds, first year graduate students will no longer teach or work in labs; instead they will take four courses each semester in their first year. Apart from graduate students, the grant supports postdocs and undergraduates. Undergraduates participate through PRUV, the undergraduate research program which got off to a strong start this summer. The grant proposal was written by Andrea Bertozzi, who now directs the program in addition to her numerous research activities. David Kraines is the director of PRUV extending his involvement with the Putnam and Modeling teams.

Several departmental officers have changed. Stephanos Venakides will take over from Tom Beale as Director of Undergraduate Studies. Xiaoying Dong will serve as Associate Director of Undergraduate Studies, a new position in the Department. Bill Allard continues as Director of Graduate Studies, while Greg Lawler replaces Bill Pardon as Associate Chair. The departmental officers, in conjunction with the staff, help the department run smoothly. All deserve lots of thanks for their commitment and the excellent jobs they do.

Students and visitors will now be able to use laptops in the building thanks to Dr. Yu and Andrew Schretter's installation of a wireless network this summer. Rooms 205 and 216 are now multimedia classrooms, each with a computer and digital projector, bringing our total of multimedia classrooms to three.

Duke's new undergraduate curriculum, *Curriculum 2000*, is being introduced this fall with the entering class. Science departments have

been encouraged to develop new courses under the rubric of *Science, Technology and Society* (STS). Joshua Holden, one of our Teaching Assistant Professors, has developed our first STS course (Mth 65S) on *Cryptography and Society*, which will consider the basics of mathematical encryption and its uses and limitations for communications in this age of information and electronic communication and commerce. Another new course is *Perspectives on Science* (Mth 61), an interdisciplinary course developed by Andrea Bertozzi that links the sciences to mathematics, statistics, and computer science.

Congratulations to Michael Reed, whose book *Methods of Modern Mathematical Physics*, which he wrote with Barry Simon, was voted *Bestseller of the XXth Century in Mathematical Physics* by the participants of the XIIIth International Congress on Mathematical Physics that took place at Imperial College, London, UK.

Finally, I'd like to alert you to several upcoming Gergen Lectures. Martin Nowak, Head of the Program in Theoretical Biology at the Institute for Advanced Study in Princeton will give a series of lectures in late September and Sir Roger Penrose will give a series of lectures in late October.

I wish you all a productive and interesting year.

—Dick Hain, Chair

## Events

### Gergen Lectures

On September 25 and 26, Martin Nowak, head of the Theoretical Biology Program at the Institute for Advanced Study in Princeton, will give three talks as part of the Gergen lecture series. Since receiving degrees in Biochemistry and Mathematics in 1989 at the University of Vienna, Nowak has written or coauthored over 160 articles on fields ranging from mathematical models of infectious diseases and virus evolution to evolutionary game theory and cooperation and the evolution of language. He will give a general lecture on "Fairness and Cooperation" at 4:15 pm on September 25 in 114 Physics, and will

talk on "The Evolution of Language," at 1:00 pm September 25 and "Virus Dynamics" on September 26 at 3:45 pm in 113 Physics.

Sir Roger Penrose, Rouse Ball Professor of Mathematics at Oxford University, will give three Gergen lectures on October 23, 24, and 25. In addition to his important work in general relativity and quantum mechanics, Penrose has made several fascinating contributions to recreational mathematics including the Penrose staircase, which served as the basis for Escher's Waterfall and Ascending and Descending prints, and non-periodic tessellations of the plane, the Penrose tilings, which later helped chemists understand quasi-crystals. His interest in artificial intelligence and consciousness is explored in his books *The Emperor's New Mind* and *Shadows of the Mind*.

John Jay Gergen was chair of the Duke Mathematics Department from 1937 until shortly before his death in 1967. Once or twice a year, distinguished mathematicians will present lectures to the community in his honor. For more information and a list of previous Gergen Lecturers, see <http://www.math.duke.edu/info/gergen.html>. ❁

## DUMU

### About DUMU

The Duke University Math Union, or "DUMU" for short, organizes activities including social events, such as picnics, movies, frisbee games with the Society for Physics Students, and our high-school math contest. Additionally, we invite speakers for the entertaining and informative Undergraduate Lecture Series. If you are interested in hearing about DUMU events and are not already on our mailing list, contact Carl Miller at [carl@math.duke.edu](mailto:carl@math.duke.edu). Read on to find out about our current plans. ❁

### The High School Math Meet

DUMU is planning to host a contest early in the Spring term for high schools in North Carolina and nearby states, and we need problems

and solutions for it. The more people who contribute, the more varied and interesting the contest will be. So, start thinking, and keep your eyes open for intriguing ideas. Math may be a tool and a subject, but it can also be a sport, a game, a kind of art, and thought-provoking fun.

We are looking for original problems of varying difficulty that can be solved in 2-10 minutes using pre-calculus mathematics only. The problems most likely to be used are those that reward insight more than experience. We will also need a few very easy problems. For examples, see last year's contest at <http://www.math.duke.edu/dumu/Problems/>.

Send submissions to [carl@math.duke.edu](mailto:carl@math.duke.edu). See <http://www.math.duke.edu/dumu> for more information about the contest.

—Carl Miller

### Competitions

The following contests are open to all undergraduates. They take place on Saturdays and are held in a math classroom in the Physics Building. If you are interested in participating in one of these competitions, or if you would just like more information, contact David Kraines at [dkraines@math.duke.edu](mailto:dkraines@math.duke.edu).

- October 28. The Virginia Tech math contest will be held from 9:00 until 11:30 in Math-Physics 120. Last year 255 students from 41 institutes participated in the 21st contest. Junior Kevin Lacker won the contest and 6 other Duke students were among the top 20 contestants. See <http://www.math.vt.edu/events/> for more information and past tests.
- December 2. The W. L. Putnam Mathematical Competition is given in two sessions of three hours each. Success in this challenging competition requires ingenuity and mathematical rigor rather than advanced mathematical knowledge. Since 1990, two Duke teams have won the competition and two have finished in second place. Last year's team came in third and Seniors John Clyde and Michael Colsher, and Junior Kevin Lacker all ranked among the top ten of the 2,900 participants. For more information, see <http://math.scu.edu/putnam/>.

## Department News

### Duke Gets VIGRE Grant

The Duke Mathematics Department has been awarded a National Science Foundation VIGRE (*Vertical Integration of Research and Education*) grant. The NSF Division of Mathematical Sciences created the VIGRE program in 1997 in order to increase interaction between undergraduates, graduates students, and postdoctoral fellows in math departments at American universities, as well as to better train these individuals to pursue a variety of careers in the mathematical sciences. There are currently VIGRE Programs at over twenty U.S. universities, all of which provide postdoctoral fellowships, graduate support and training, and undergraduate research opportunities.

The Duke VIGRE program offers graduate students the opportunity to join one of several interdisciplinary research groups in mathematically related areas. The Grant supports a number of postdoctoral fellows and graduate students at Duke. The first VIGRE postdoc, John Mathews from NC State, is working with David Schaeffer on granular flow. Fourteen graduate students currently receive full or partial support from the VIGRE grant.

—Juliet Pulliam

### PRUV Fellows

The undergraduate component of Duke's VIGRE Program was launched this summer with a pilot version of the PRUV (Practical Research for Undergraduates with VIGRE) Program. The PRUV Program matches undergraduates who are going into their third or fourth year with faculty mentors. Through the mentor relationship, the students plan and carry out original research projects, on which they will base their senior theses. The students currently participating in the PRUV Program, Michael Colsher, Sam Malone, Carl Miller, and Daniel Neill, discussed their summer work at seminars on September 18 and 21.

Applications for the 2001 PRUV Fellowships are currently available. For further infor-

mation, please contact Dr. David Kraines at [dkrain@duke.edu](mailto:dkrain@duke.edu) or see <http://www.math.duke.edu/vigre/pruv.html>.

—Juliet Pulliam

### Online Learning Materials

Professor Lang Moore is the principal investigator for a new NSF-funded project of the Mathematical Association of America. The goal is to create a national digital library of online learning materials consisting of a new online MAA journal, the *Journal of Online Mathematics and its Applications* (JOMA), a library of online mathematics learning materials, and a site where all commercial learning materials (books, software, etc.) for undergraduate mathematics will be described and reviewed. The online material should assist faculty members around the world in designing new courses. Professor David Smith is the first editor of JOMA.

John Michel, former chair of mathematics at Marietta College (Ohio), is visiting Duke University this fall. He is working with David Smith and Lang Moore on material for the Connected Curriculum Project, a set of interactive learning materials for mathematics and its applications funded by the National Science Foundation. See <http://www.math.duke.edu/education/ccp/> for details. ❁

### New Faculty

The following professors and research associates have joined the department this fall.

- **Eddie Fuller** (PhD U Georgia), Lecturing Fellow. *Geometry and scientific computing*.
- **Ning Ju** (PhD Indiana U), Assistant Research Professor. *Applied mathematics and numerical analysis*.
- **Mariano Marcano-Velazquez** (PhD SUNY at Stony Brook), Visiting Assistant Professor. *Mathematical biology*.
- **John Matthews** (PhD NC State), Research Associate. *Applied mathematics, granular flow*.
- **Owen Patashnick** (PhD U Chicago), Assistant Research Professor. *Algebraic geometry*.

- **Andreas Rosenschon** (PhD U Maryland), Visiting Assistant Professor. *Algebraic geometry*.
- **Padmanabhan Sundar** (PhD Purdue), Visiting Professor. *Probability*.
- **K. Swetketu Virbhadra** (PhD Physical Research Laboratory in India), Research Associate. *General relativity and astronomy*.

### Other News

To read about other news, honors and events concerning mathematics at Duke, visit [www.math.duke.edu/news/](http://www.math.duke.edu/news/). The on-line calendar at [www.math.duke.edu/cgi-bin/mcal](http://www.math.duke.edu/cgi-bin/mcal) lists both regular and special seminars and colloquia for the upcoming weeks. The department maintains video archives of talks, lecture series and special conferences at Duke many of which are available on-line. See [www.math.duke.edu/computing/broadcast.html](http://www.math.duke.edu/computing/broadcast.html) for more information.

## Awards

### “Bestseller of the XXth Century”

*Methods of Modern Mathematical Physics* by Duke professor Michael Reed and Barry Simon of Cal Tech was voted “the Bestseller of the XXth Century in Mathematical Physics” by the participants of the XIIIth International Congress on Mathematical Physics. Published in four volumes during the 1970’s, this book was deemed to be the one with the greatest influence on mathematical physics over the previous 100 years. See <http://icmp2000.ma.ic.ac.uk/> for details. ❁

### Layton awarded NIH Grant

The National Institute of Health has renewed its grant to associate professor Harold Layton for his work on mathematical models of renal dynamics. The \$600,000 grant will fund “theoretical studies on tubuloglomerular feedback, the renal microvasculature, and the urine concentrating mechanism” at Duke and SUNY Stony Brook through June 2004. ❁

## News From The Graduate Program

### New Graduate Students

- David Anderson, University of Virginia
- Robert Buckingham, MIT
- John Cain, Rutgers University
- Benjamin Cooke, MIT
- Carina Curto, Harvard University
- Andrew Feist, Central Missouri State University
- Daniel Fox, Hampshire College
- Daniel Goldstein, University of North Carolina
- Ryan Haskett, Harvey Mudd College
- Kwok-Man Hui, Purdue
- Robert Karp, University of Cincinnati
- Thomas Laurent, Paris VII
- Timothy Lucas, Occidental College
- Philipp Mathey, ETH, Zurich
- Wenjun Ying, Tsinghua University

### Graduate Awards

At the first Departmental meeting of the year, the LP and Barbara Smith Teaching Award for the 1999-2000 academic year was presented to Ted Welsh. Ted was recognized for his many years of dedication to teaching mathematics, his innovative ways of getting student feedback in calculus, and his contributions to curriculum development. Ted has done an outstanding job in the classroom, and this fall he became only the second graduate student to supervise a multi-section course for our department.

The substantial monetary prize that accompanies this award was made possible by Captain L.P. Smith and Mrs. Barbara Smith. Capt.

Smith was the Supervisor of First-year Instruction from 1973 until 1982, when he retired for the second time from a full-time career. There is a complete list of previous winners at the web site [http://www.math.duke.edu/first\\_year/lpsmith.html](http://www.math.duke.edu/first_year/lpsmith.html). Our congratulations to Ted on this outstanding achievement.

—Lewis Blake

## New Courses

### ADVANCE

ADVANCE is a new program for first-year women who are interested in mathematics and its application to the quantitative sciences. Students participating in the ADVANCE Program take 4.5 credits, including a half-credit course each semester entitled 'Perspectives on Science,' taught by Professor Andrea Bertozzi. This course features a series of lectures given by women who are involved in exciting research relating to a wide variety of quantitative fields, including toxicology, economics, genomics, and biomechanics. ADVANCE students will work in groups to design web-pages about the speakers, their research, and the history and societal impact of their fields. The project will teach the students how quantitative methods can be applied to real research problems while increasing their familiarity with the role models that they meet. In the Fall, the course will focus on applications of mathematics, computer science, and statistics to medicine, biology, and environmental science. In the Spring, the focus will be on applications to engineering and the physical and social sciences.

In addition to the 'Perspectives on Science' course, ADVANCE students will take quantitative reasoning courses each semester, a specially-designed version of the freshman Academic Writing course in the Fall, and thematically-designed first-year seminars in the Spring.

For further information, please visit the ADVANCE web site at [www.aas.duke.edu/advance](http://www.aas.duke.edu/advance) or contact Dr. Andrea Bertozzi at [bertozzi@math.duke.edu](mailto:bertozzi@math.duke.edu).

—Juliet Pulliam

### Math 65S

Math 65S is an introduction to basic ideas of modern cryptography with emphasis on implementation, applications in daily life, and implications for the individual and society. Topics covered include: the history of cryptography and cryptanalysis, public and private key cryptography, digital signatures, limitations of modern cryptography, applications to electronic communications and electronic commerce, privacy, computer security, and law enforcement. Related ethical questions will be considered including the debate over personal privacy versus public security.

—Joshua Brandon Holden

### Problem Corner

#### Solutions from Last Issue

##### Problem 1: Integral Inquiry

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function such that  $\int_{t=0}^1 f(t) dt = 1$  and  $f(x+1) = f(x)$  for all  $x$ . Show that for some real value  $x_0$ ,  $\int_{t=x_0}^{x_1} f(t) dt \geq x_1 - x_0$  for all  $x_1 > x_0$ .

##### Solution to Problem 1:

Note first that  $f(t+n) = f(t)$  for any  $n \in \mathbb{Z}$ , so,  $\int_x^{x+1} f(t) dt = \int_x^{\lceil x \rceil} f(t) dt + \int_{\lceil x \rceil}^{x+1} f(t) dt = \int_{x-\lceil x \rceil+1}^1 f(t) dt + \int_0^{x-\lceil x \rceil+1} f(t) dt = \int_0^1 f(t) dt = 1$  for any  $x$ .

Define  $F: \mathbb{R} \rightarrow \mathbb{R}$  by  $F(x) = \int_{t=0}^x f(t) dt - x$ . With  $\int_0^x f(t) dt = -\int_x^0 f(t) dt$  if  $x < 0$ . Since  $F$  is continuous,  $F([0, 1])$  is compact so there exists a minimum value  $F(x_0)$  for  $x_0 \in [0, 1]$ .  $F$  is periodic:  $F(x+1) = \int_0^{x+1} f(t) dt - (x+1) = \int_0^x f(t) dt + (\int_x^{x+1} f(t) dt - 1) - x = \int_0^x f(t) dt - x = F(x)$ , thus  $F(x_0)$  is in fact a minimum over all values of  $F(x)$ . Therefore for any  $x_1 \in \mathbb{R}$ ,  $\int_0^{x_1} f(t) dt - x_1 \geq \int_0^{x_0} f(t) dt - x_0 \Rightarrow \int_0^{x_1} f(t) dt - \int_0^{x_0} f(t) dt \geq x_1 - x_0 \Rightarrow \int_{x_0}^{x_1} f(t) dt \geq x_1 - x_0$ .

##### Problem 2: Are you dense?

Define sequences  $(a_n)$  and  $(b_n)$  by  $a_0 = 0$ ,  $b_0 = 1$ ,  $a_n = a_{n-1} + 3b_{n-1}$ ,  $b_n = b_{n-1} - 3a_{n-1}$ . Show

that the set  $\{a_n/b_n : b_n \neq 0\}$  is dense on the real line.

##### Solution to Problem 2:

We show inductively that  $(1+3i)^n = b_n + ia_n$ . The base case is obvious, and  $(1+3i)^{n+1} = (b_n + ia_n)(1+3i) = (b_n - 3a_n) + (a_n + 3b_n)i = b_{n+1} + ia_{n+1}$ .

When  $b_0 \neq 0$ ,  $a_n/b_n = \tan(\text{Arg}[b_n + ia_n]) = \tan(\text{Arg}[(1+3i)^n]) = \tan(n \text{Arg}[1+3i] \bmod 2\pi)$ , where  $(x \bmod y)$  denotes the unique real value  $r \in [0, y)$  such that  $x - r$  is an integral multiple of  $y$  and  $\text{Arg}(z)$  is the argument polar angle of  $z$  ( $0 \leq \text{Arg}[z] < 2\pi$ ). It is a well-known fact that the values  $\{nx \bmod y : n \in \mathbb{Z}\}$  are dense in the interval  $[0, y)$  if and only if  $x/y$  is irrational. From this it would follow that  $\{n \text{Arg}[1+3i] \bmod 2\pi : n \in \mathbb{Z}\}$  is dense in  $[0, 2\pi)$ , and therefore by continuity  $\{\tan(n \text{Arg}[1+3i] \bmod 2\pi) : n \in \mathbb{Z}\} = \{a_n/b_n : n \in \mathbb{Z}\}$  is dense in  $\mathbb{R}$ . So to finish the proof we need only to show that  $\text{Arg}[1+3i]/2\pi$  is irrational.

Suppose not. Choose the smallest positive integer  $n$  such that  $n \text{Arg}[1+3i] \bmod \pi = 0$ ; then

$$(1+3i)^n \in \mathbb{R} \Rightarrow (1+3i)^n = \pm |1+3i|^n = \pm \sqrt{10}^n \Rightarrow \left(\frac{-3+4i}{5}\right)^n = \left(\frac{1+3i}{\sqrt{10}}\right)^{2n} = (\pm 1)^2 = 1.$$

So  $(-\frac{3}{5} + \frac{4}{5}i)$  is a primitive  $n$ th root of unity. Yet we know from the theory of cyclotomic fields that the minimal  $\mathbb{Q}$ -polynomial of a primitive  $n$ th root of unity is of degree  $\phi(n)$ , while  $(-\frac{3}{5} + \frac{4}{5}i)$  satisfies a quadratic polynomial, so we must have  $\phi(n) = 2 \Rightarrow n = 4$ , which is clearly untrue. The proof is complete.

#### New Problems

##### Problem 1: Party Preparations

A party host knows that either  $p$  people or  $q$  people will be attending his party. He wishes to cut the cake ahead of time in such a way that, whether  $p$  guests or  $q$  guests attend, he can divide the cake evenly among the guests without any further cutting. What is the least number of pieces into which the cake must be cut in order to accomplish this? [Leningrad Olympiad]

**Problem 2: Graph Counting**

Let  $O(n)$  denote the number of connected labeled graphs on  $n + 1$  vertices with an odd number of edges, and let  $E(n)$  denote the number of connected labeled graphs on  $n + 1$  vertices with an even number of edges. Prove  $E(n) - O(n) = (-1)^n n!$ . [Jacob Lurie]

**Your Editors**

Many thanks to Juliet Pulliam '02. As new editor of the DMN, she has contributed several articles and significantly improved and expanded our paper. DUMU president and puzzle editor, Carl Miller '01, continues to provide challenging problems for you to work on. He welcomes your solutions to these and your suggestions for problems for the DMN and for the up-coming high school Duke Math Meet. We are all very happy that Staff Specialist, Janet Stockburger, our first Production Manager, has put together the DMN so professionally and in record time.

We welcome suggestions for features for future issues. Please share with us news, events, honors and awards of a mathematical nature. Send items to Juliet Pulliam [jrp10@duke.edu](mailto:jrp10@duke.edu).

—David Kraines, DMN Faculty Sponsor

Student Editor  
Juliet Pulliam ..... [jrp10@duke.edu](mailto:jrp10@duke.edu)  
Problem Editor  
Carl Miller ..... [carl@math.duke.edu](mailto:carl@math.duke.edu)  
Faculty Sponsor  
David Kraines ..... [dkraines@math.duke.edu](mailto:dkraines@math.duke.edu)  
Production Manager  
Janet Stockburger ..... [jstock@math.duke.edu](mailto:jstock@math.duke.edu)  
  
Department of Mathematics  
Box 90320  
Durham, NC 27708-0320  
[http://www.math.duke.edu/math\\_news/](http://www.math.duke.edu/math_news/)