Duke University Math News

February 2, 1999

Notes from the DUS

To Mathematics Majors and Minors,

Welcome back for the new semester. A special welcome to those who are new majors. If you are a new first major in math, we will be assigning you an advisor before registration for the fall, but in the mean time feel free to ask me and other faculty for advice. Also, much information can be found at the department web site, and especially in the *Handbook for Mathematics Majors and Minors*, available at the web site www.math.duke.edu as well as in "hard copy."

Many of you are probably thinking about what you will be doing during the summer. I will remind you that we have a collection of information about study in math and math-related fields as well as some information about job opportunities in the outer area of the department office, 121 Physics Building. There you will find mailings we have received recently about actuarial science, graduate study, fellowships, summer programs and employment opportunities. A notebook labeled "Summer Opportunities" has a more extensive collection from past years.

—*Tom Beale*

Opportunities

Research Experience for Undergraduates

The National Science Foundation sponsors Research Experiences for Undergraduates at a score of sites around the country. These 6 to 8 week programs offer a substantial stipend as they provide opportunities for students to do original research. To find out about them, look first at the department web site on the page for "The Undergraduate Program." Under "Mathematics Resources at Other Sites" you will see a link to the NSF list of REU's (www.nsf.gov/mps/dms/reulist.htm). There is also a link to a set of reviews by the Harvard Math Club (www.math.harvard.edu/~mathclub/reu).

DMN Seeks New Student Editor

Since I am graduating this May, *Duke Math News* will need a new student editor. The position involves collecting articles from faculty and students, editing them, and typesetting them with LaTeX. It takes a few hours three or four times a year and the pay is good. It's a great way to meet people and get involved in the department. If you are interested, please contact me (wgm2@acpub.duke.edu) or Dr. Kraines (dkrain@math.duke.edu).

—*Garrett Mitchener*

Events and Contests

Duke Dominates in Virginia Tech Math Contest

At 8:30 a.m. on Saturday morning, October 31, 186 undergraduates from 34 colleges and universities throughout the Southeast competed in the 20th annual Virginia Tech Math Contest. Students from Duke University swept the field, capturing first, second and third place, six of the top seven and nine of the top fourteen spots. Winning the contest was Andrew Dittmer '99 with Carl Miller '01 second and John Clyde '01 third. Other Duke contestants ranking in the top 10% were Nathan Curtis '01, Jeff Mermin '00, Kevin Lacker '02, Andy Goss '02, Sarah Dean '00 and Craig DeAlmeida '99. In addition, Christopher Beasley '99, Nathan Bronson '98, Daniel Neill '01, Spencer Shepard '00, and Tristan Tager '02 each finished among the top third. Dittmer, Miller, Clyde and Curtis each received a cash prize. The recent Duke performance exceeds even that in 1997 when Duke students took 8 of the top 16 slots.

High School Math Meet a Success

On Saturday, November 14, about 100 high school students from North and South Carolina participated in the Duke Math Meet, organized by the Duke University Math Union (DUMU) and the problem solving seminar. Top place in the ARML style team competition was captured by the North Carolina School of Science and Mathematics with the South Carolina Governor's School of Science and Mathematics placing second. Enloe High School, Chapel Hill High School and the Charlotte Math Club rounded out the top five. After a long series of tie-breakers, Daniel Wong from NCSSM beat out Ioana Gradinaru from SCGSSM. Danny Morano and Daniel Wilson from NCSSM and Alexander Chu from Chapel Hill took third, fourth and fifth places respectively.

DUMU plans to hold the next Duke Math Meet in October, 1999. Those of you who come from the southeast should encourage your high-school math teacher to organize a team. DUMU welcomes suggestions for problems and seeks volunteers to help with the organization of the next competition.

Mathematical Contest in Modeling

The MCM, sponsored by the Society for Industrial and Applied Mathematics and other organizations, is a contest for undergraduates dealing with real-world problems and solutions. The university is allowed to enter several teams of three students. Each team works independently to solve one of the problems posed and, during the course of the weekend, writes a paper about the solution. The papers are judged and given a ranking, either Outstanding, Meritorious, Honorable Mention, or Successful Participant. The Outstanding papers are published in the fall issue of *The UMAP Journal*. Anyone interested in applied math should find the MCM worthwhile.

Last year, the Duke team won an Outstanding rating for the first time ever. The three team members were flown to Toronto, Canada, to present their solution to the grade inflation problem at Math Fest.

Starting this year, there is a new, interdisciplinary category. According to the rules, the problem will be about pollution and will require knowledge of chemistry, environmental science, and data analysis.

Good teams have a mixture of mathematical talent, writing ability, and computer skills. Expect to do some reading and research. Computer simulations are frequently useful, as are programs such as Maple and MathCad. Since there is no right answer, no specific knowledge is required; good solutions usually come from creatively applying what you already know to the problem. However, it is sometimes useful to explore unfamiliar subjects. For example, a problem from several years ago involving a computer network was best solved using some advanced but learnable graph theory. For the new interdisciplinary category, at least one of the team members should have some expertise in the area of the problem.

The contest does take up a lot of time, and most of it is spent with other members of the team. This year, the contest goes from midnight on Friday, February 5 until 5:00 p.m. on Monday, February 8.

Contact Garrett at `wgm2@acpub.duke.edu` as soon as possible if you are interested.

Programming Team Advances to Finals

On November 14, the Computer Science Department hosted the 30 teams from the local area to compete in the Regional ACM Programming Contest. Over 160 teams from universities and colleges throughout the region competed against one another in this day-long event. At the end of the day, all five Duke teams, coached by Nate Bronson, had finished in the top 30 and one placed fifth overall. John Clyde, Andrew Dittmer, and Meetesh Karia will travel to compete in the International Finals held in the Netherlands in April of 1999. Professor Susan Rodger acted as the site director. Both faculty and staff volunteered their offices for the teams to use, and Beatrice Chestnutt and Diane Riggs volunteered their weekends to help feed approximately 90 hungry college students. For a complete summary of the final results visit `http://midatl.cslab.vt.edu`

Publications

Math Horizons Now Available

Several issues of the quarterly magazine *Math Horizons* are available first come first serve in room 121 Physics.

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*I*t can be of no practical use to know that π is irrational, but if we can know, it surely would be intolerable not to know.

—E. C. Titchmarsh

Department News

Research Intensive Courses for Curriculum 2000

The recent adoption of Curriculum 2000 by the Duke Arts and Science Council will require some significant changes in our undergraduate program over the next few years. After a phase-in period, each Duke student must complete two Research-Intensive (RI) exposures, at least one of which must be in the student's major. The goal of this requirement is to develop in students "an understanding of the process by which new knowledge is created, organized, accessed, and synthesized. It also fosters a capacity for the critical evaluation of knowledge and the methods of discovery. Engagement with the research process better prepares not only undergraduates who wish to pursue further study at the graduate level but also those who seek employment in a rapidly changing and competitive marketplace."

Although some students produce a substantial paper qualifying for graduation with distinction, most majors do not have the time to devote to such research. One alternative would be to add a half-credit research preceptorial to many of our math seminars and a few other courses at the junior-senior level. Since most majors take two or three such courses, they will have considerable flexibility as to which of their course

to expand into their RI exposure. Another option might be interdisciplinary research seminars team-taught with professors in economics, computer science, and physics.

Current undergraduates are not directly affected by Curriculum 2000, but your experience and opinions on this subject could help the department develop appropriate options. Have you met or will you meet the RI goals of Curriculum 2000? Would you have taken an RI preceptorial were it available? If so, for which course and with which professor? What other avenues should the department explore to help students satisfy this requirement? Please talk over your ideas with your classmates and with your professors or send your comments to Dr. Kraines at dkraines@math.duke.edu.

Course on Financial Math for Fall

MTH 238 *Topics in Applied Mathematics: Mathematics of Finance*

Methods for pricing derivatives and other financial instruments have changed considerably since the papers of Black and Scholes and Merton in the early seventies. One of the key mathematical tools is stochastic calculus—integrals and differential equations including terms from random processes such as Brownian motion. This course will introduce stochastic calculus and show how it is used in pricing financial instruments. Prerequisites: Math 135 and 104 or the equivalent.

The course will be taught in the fall 1999 semester by Dr. Lawler.

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*F*ourier is a mathematical poem.
      —William Thompson, Lord Kelvin

Course on the Physics and Mathematics of Light

MTH 196S *Mathematics of Light Deflection in the Universe*

The course begins with Einstein's study of light deflection by the gravitational field of a point mass (e.g., star) and concludes with the case

of arbitrary weak-field deflectors (e.g., galaxies). Caustics, image counting, and magnification are treated in detail. Students are introduced to some basic ideas and techniques from differential geometry, complex analysis, Morse theory, and Whitney's singularity theory. This course is oriented towards math and (theoretical) physics majors. It exposes students to an arena where pure mathematics, theoretical astrophysics, and observational data converge. Prerequisites: Math 103 and 104. Familiarity with basic introductory physics is useful, but not mandatory.

The course will be taught in the fall 1999 semester by Dr. Petters as a seminar and will become a regularly-offered course as part of Curriculum 2000.

Problem Corner

Solutions from Last Issue

Solution to Problem 1:

Pick two lines on the top of the board and two lines on the left side and use all the squares in between to form a rectangle. There are a total of $\binom{9}{2}$ ways to pick each pair of lines for a total of $36^2 = 1296$ rectangles. For squares, we pick one vertical line and one horizontal line, plus n squares down and right, so we get $(9-n)(9-n)$ squares of size n for a total of $1+4+9+\cdots+49+64 = 204$ squares.

Solution to Problem 2:

Note: a and b are positive integers throughout.

Note that $a \leq S_b$ implies $S_a \geq S_{S_b}$ (because S is non-increasing) which implies $b \leq S_{S_b} \leq S_a$. An identical argument shows that $b \leq S_a$ implies $a \leq S_b$. Thus, $a \leq S_b$ if and only if $b \leq S_a$.

Consider a bar graph in which the unit block whose upper right-hand corner lies at (a,b) is included in the graph if and only if $b \leq S_a$. We can determine sums of the sequence $\{S_n\}$ by counting blocks in this graph. The rule $a \leq S_b \iff b \leq S_a$ implies that the graph is symmetric around the line $x = y$.

Choose m to be the largest integer such that (m,m) is included in the graph (that is, the largest integer such that $m \leq S_m$). We can divide the graph into three parts around the point (m,m):

$$A = \{(a,b) : a \leq m, m < b \leq S_a\}$$
$$B = \{(a,b) : a \leq m, b \leq m\}$$
$$C = \{(a,b) : m < a, b \leq S_a\}$$

Observe that $(a,b) \in A \iff b \leq S_a$ and $b > m \iff a \leq S_b$ and $b > m \iff (b,a) \in B$. Thus sets A and C are mirror images of each other around the line $x = y$. So $|A| = |C|$, and hence

$$\sum_{k=1}^{m} S_k - \sum_{k=m+1}^{n} S_k = (|A| + |B|) - |C| = |B| = m^2.$$

New Problems

Problem 1: *Sixty-Four Derivatives*

A function $f : \mathbf{R} \to \mathbf{R}$ is continuous at zero and satisfies $f\left(x^2\right) = f(x) - x$ for every $x \in (-1,1)$. Find the 63rd and 64th derivatives of f at zero.

Problem 2: *No Such Luck*

Show that there does not exist a doubly-infinite sequence of positive real numbers which, for every integer i, satisfies

$$2x_{i-1}x_{i+1} > x_{i-1}x_i + x_ix_{i+1}.$$

("Doubly-infinite" means that the sequence proceeds infinitely in both directions: $\ldots, x_{-1}, x_0, x_1, x_2, \ldots$)