

Duke University Math News

October 2, 2003

Welcome

As the Director of Undergraduate Studies in Mathematics, I would like to welcome you back to classes. I hope that you had a productive, or an interesting, or a fun summer (for myself, I had all three). We have a very lively environment at Duke for learning mathematics; even so, I welcome any suggestions that you may have to make it even better.

The 2003-2004 edition of the *Handbook for Mathematics Majors and Minors* is now available, and you can pick up a copy in the Mathematics Office, 121 Physics. The Handbook is also available at the web site of the department <http://www.math.duke.edu> under the heading The Undergraduate Program.

Registration for the spring semester begins on Wednesday, October 29; advising begins on Monday, October 27. If your first major is mathematics, you will have an advisor in the Mathematics Department. For those of you who recently declared a first major in mathematics, I will assign you an advisor soon. If you are not a first major, your advisor will be in the department of your first major, but please feel free to talk to me about your selection of mathematics courses and the requirements for the mathematics major. hodel@math.duke.edu

Here is a preview of courses above MTH 104 that will be offered next spring: 111 (three sections), 114 (three sections), 121, 126, 128S, 131 (two sections), 132S, 133, 135 (two sections), 136, 139, 160S, 201, 204, 206, and numerous other 200 level courses.

—Richard Hodel
Director of Undergraduate Studies

Undergraduate News

DUMU

The Duke University Math Union, DUMU, organizes activities such as picnics, movies, frisbee contests, tutoring at local high schools and the Duke Math Meet (see below). DUMU also hosts world-renowned mathematicians for the Undergraduate Lecture Series. If you would like to hear about DUMU events, contact Oaz Nir at on5@duke.edu.

High School Math Meet

The annual ARML style math meet for high school students in North Carolina and the Southeast will be held on Saturday, November 15, from 10 to 5. Duke students invite teams, construct and grade the problems, and present the awards. Those of you who went to high school in the region should encourage your former math teacher to send a team. Volunteers are needed to help organize the meet and to compose original problems at the pre-calculus level, especially clever problems with easy solutions. Contact Paul Wrayno at pmw5@duke.edu if you can volunteer to help or to submit solutions. See <http://www.math.duke.edu/dumu> for more information.

Competitions

The following competitions are open to all regularly registered undergraduates. There is no cost, lunch is provided and monetary prizes are awarded. If you are interested in participating or if you would just like more information, contact David Kraines at dkraines@math.duke.edu.

The Virginia Tech Regional Math Contest will be held from 9:00 to 11:30 on Saturday, November 1. Last year, 248 students from 41 schools participated including 17 from Duke. Of the top 18 participants, 10 were Duke students. See <http://www.math.vt.edu-events/> for information and past exams.

The William Lowell Putnam Mathematical Competition is a far more challenging event. It will be held on Saturday December 6 from 10 to 1 and 3 to 6. Last year a total of 3349 students from 476 colleges and universities in Canada and the United States participated in this six hour competition for individual and team awards. The Duke team of David Arthur, Oaz Nir, and Melanie Wood placed third with Wood being named a Putnam Fellow for her rank among the top five individuals. Since 1990, a team from Duke has won the Competition three times, placed second twice and third three times.

PRUV Fellows

The National Science Foundation supports a research program for Duke juniors with stipends of \$3000. The heart of the program is six weeks of intensive research during the summer under the direction of a Duke professor. Student research papers are expected to qualify for Graduation with Distinction.

The current PRUV Fellows are, David Arthur, Suzy Borgschulte, Lauren Childs Ryan Letchworth, Lori Peacock, Lindsay Piechnik, Matthew Touns, and Jenna VanLiere. For more information and an application form, see <http://www.math.duke.edu/vigre/pruv/>.

Faculty Scholars

Math major David Marks and Computer Science and Math major Ethan Eade were two of the three seniors named as Faculty Scholars, the highest honor awarded by Duke faculty to undergraduates. Students are selected on the basis of GPA, independent research, potential for innovative scholarship and an intention to pursue a scholarly career. Since 1991, 12 math majors have been named Duke Faculty Scholars.

Graduate Program News

New Graduate Students

- **Rann Bar-on**, University of Warwick-UK
- **Sergei Belov**, St. Petersburg State-Russia
- **Paul Bendich**, Grinnell College
- **Mihaela Guberovic**, Virginia Military Institute
- **Wai J. Law**, Georgia Institute of Technology
- **Abraham Smith**, University of Wisconsin-Madison
- **Matthew Surles**, University of Delaware
- **Feng Xu**, Tsinghua University

Graduate Courses

New and Topics courses for Fall 2004

1. **Math 283 *The Mathematics of Photonic Crystals*** (Stephanos Venakides). The course will examine wave propagation in spatially periodic systems. Topics that will be covered include the Maxwell and Helmholtz equations, the integral formulation of the theory of harmonic fields via Calderon projections, the theory of Floquet Bloch waves, defects and resonance in photonic crystals, and (time permitting) nonlinear behavior.
2. **Math 277 *Toric Varieties in Math, Physics and Computation*** (Christian Haase). We will study the fascinating interplay between the algebraic geometry of toric varieties and the combinatorics of convex polytopes/polyhedra. A previous encounter with (complex) projective space is advantageous. There will be one or two organizational meetings in the Fall. After studying the basics of the subjects, students will prepare a talk and write a report on a more

specialized "pure math" subject such as convex polytopes, algebraic geometry, commutative algebra, or string theory or a more applied topic such as solving systems of polynomial equations, integer programming, and the like. These talks may be held at some pleasant location during a weekend in April.

Students can find out more about these and other graduate courses being offered this term at http://www.math.duke.edu/graduate/grad_courses.html.

—John Trangenstein, Director of Graduate Studies

L.P. and Barbara Smith Award

This year's winners of the L. P. and Barbara Smith Award for Excellence in Teaching are Chris Hale, Michael Kozdron, and Christian Benes. Chris, Michael, and Christian have all done an excellent job, over many semesters, teaching calculus in the first-year program. Their success has been evident from their students' successes and from the positive responses from their students about the quality of their teaching. Each of these winners will receive a check in the amount of \$1500 from the fund established in 1981 by Captain L. P. Smith and Barbara Smith. In addition to the three primary winners, Daniel Goldstein was awarded a check for \$500 in recognition of his recent significant accomplishments in teaching.

L. P. Smith, an aviator in the US Navy who retired with the rank of Captain, served as Supervisor of First-year Instruction in the Mathematics Department from 1973 until his (second) retirement in 1982. Smith's goal was to reward those graduate students who work hard, have demonstrated a long-term commitment to teaching and whose teaching has reached a consistent level of excellence. The Mathematics Department is grateful to these graduate students and to the Smiths for making this award possible.

Faculty News

New Faculty

The following professors and research associates have joined the department this fall.

- **Andrew Comech** (PhD Columbia University), Visiting Assistant Professor. *Partial Differential Equations*.
- **Jonathan Hanke** (PhD Princeton University), Assistant Research Associate. *Number Theory and Algebraic Geometry*.
- **Hyung Ju Hwang** (PhD Brown University), Assistant Research Professor. *Dynamical instabilities of fluids, regularity for kinetic equations*.
- **Garrett Mitchener** (PhD Princeton University), Research Associate (VIGRE). *Dynamical Systems, ergodic theory, biological models*.

Faculty Advisor for DUMU

Dr. Garrett Mitchener, the editor of the Duke Math News during each of his four years as an undergraduate at Duke, has joined our faculty as a Research Associate.

As president of DUMU from 1997 to 1999, Garrett helped organize the Duke Math Contest for high school students throughout the southeast. He participated in the Putnam and VPI contests and graduated with high distinction for his senior thesis "Lattices and Sphere Packing".

Garrett excelled in the annual Mathematical Contest in Modeling leading two teams to Meritorious ratings and two teams to Outstanding ratings. He was a B. M. Goldwater Scholar and winner of the Julia Dale Prize for excellence in mathematics.

After graduating from Duke in 1999, Garrett attended Princeton University in Applied and Computational Math. In his PhD dissertation under Professor Martin Nowak, he studies a mathematical model of human language based on an ecological model. This model represents a

population of individuals that interact and learn languages from their parents. Their survival depends on their ability to communicate with the population as a whole. The model can be used to study the circumstances that lead to a coherent population, where everyone uses the same language, or to an incoherent population, where multiple grammars coexist, or to other behaviors, such as oscillations and chaos.

Mitchener will be faculty advisor for DUMU and will help coordinate the Duke Math Meet scheduled for Saturday November 15. Please contact Garrett Mitchener at wgm@math.duke.edu if you would like more information about DUMU events.

Problem Corner

New Problem Corner Editor

First year student Nikifor Bliznashki has become the new Problem Corner Editor for the Duke Math News. A native of Sofia, Bulgaria, Nikifor won many medals in national and international math competitions. His first of what we hope will be many sets of math problems are listed below. Do give them a try. Our thanks to David Arthur for his many interesting problems over the past few years. We wish him the best as he prepares for graduate school.

Solutions from Last Issue

- Problem.** At a party, 25 mathematicians go and shake hands with each other. Show that after everything is done, one of the mathematicians must have shaken hands with an even number of people.

Solution. Let n_i denote the number of hands that the i th mathematician shakes. Since each handshake is shared by exactly 2 mathematicians, we know that $\sum_{i=1}^{25} n_i$ gives twice the total number of handshakes, which is even. On the other hand, the sum of 25 odd numbers is odd, so it follows that n_i is even for some i .

- Problem.** Find all pairs of positive real numbers, x and y , satisfying $\frac{x}{x+1} + \frac{y}{y+1} = \frac{x+y}{x+y+1}$.

Solution. Note that $\frac{x}{x+1} \geq \frac{x}{x+y+1}$, and that $\frac{y}{y+1} \geq \frac{y}{x+y+1}$, with equality occurring in both cases iff either x or y is 0. Adding these, we find $\frac{x}{x+1} + \frac{y}{y+1} \geq \frac{x+y}{x+y+1}$ with equality iff either x or y is 0. Thus, the general solution is $(t, 0)$ and $(0, t)$ for $t \geq 0$.

- Problem.** Consider a polyhedron with at least five faces such that exactly three edges emerge from each of its vertices. Two players play the following game:

Each player, in turn, signs his or her name on a previously unsigned face. The winner is the player who first succeeds in signing three faces that share a common vertex.

Show that the player who signs first will always win by playing as well as possible.

Solution. Let v , e , and f denote the number of vertices, edges and faces respectively of the polyhedron P . By Euler's formula, $v - e + f = 2$. Since each vertex is the end of 3 edges and each edge has two vertices, $v = \frac{2}{3}e$. If each face had exactly 3 edges, then $f = \frac{2}{3}e$ and $(\frac{2}{3} - 1 + \frac{2}{3})e = \frac{1}{3}e = 2$ so $e = 6$ and P would have only 4 faces. Since P has more than 4 faces, at least one face must have at least 4 edges. If the first player signs this face on his first turn, it is not hard to see that he can force a win in 3 moves.

New Problems

- Assume that the opposite angles of a convex quadrilateral sum to 180° . Show that the quadrilateral can be inscribed in a circle.
- (Bulgarian Olympiad) Let x_1, x_2, x_3, x_4 , and x_5 be real numbers. Find the smallest natural number n with the following property: if any n different sums $x_p + x_q + x_r$, $1 \leq p \leq q \leq r \leq 5$, are equal to zero, then $x_1 = x_2 = x_3 = x_4 = x_5 = 0$.
- Prove that the equation $y^2 = x^5 - 4$ has no solution in natural numbers.

Submit solutions or suggestions for new problems to Problem Editor Nikifor Bliznashki nb22@duke.edu

Duke Math News

The *Duke Math News* is published several times a year and is distributed to those in the Duke mathematics community by campus mail. For previous editions and other news, see www.math.duke.edu/news/. We welcome items of interest for our next issue. Send them to jones@math.duke.edu or dkrain@duke.edu

To read about other news, honors and events concerning mathematics at Duke, visit www.math.duke.edu/news/. The on-line calendar at www.math.duke.edu/mcal lists both regular and special seminars and colloquia for the upcoming weeks. The department maintains video archives of talks, lecture series and special conferences at Duke, many of which are available, on-line. See www.math.duke.edu/computing/broadcast.html for more information.

—David Kraines, DMN Faculty Sponsor

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