Math 26L
Final Exam
April 30, 2002

NAME:____

INSTRUCTOR:______________________ SECTION: __________

Instructions:

This exam has ten pages in addition to this cover sheet. For grading purposes the point value of the entire exam is 200 points. The values of the individual questions are indicated beside the questions. We suggest that you spend not more than 15 minutes on each page. If you follow this strategy for time allocation, then you will have 30 minutes at the end to use for checking, refinement, and correction.

Some of the exam questions include carefully worded descriptions or directions. In order to complete the problems correctly, you may need to read these instructions carefully before attempting the problems. You may use your calculator and one sheet of notes that you prepared for this exam. ***You must show your work to receive credit****. In particular if you use the calculator to assist in answering a question, be sure to reproduce (by hand) any relevant graphs and explain any relevant mathematics.*

When you turn in your exam, be sure that each sheet has your name and section number on it and that you have signed the honor code below.

Honor Code:

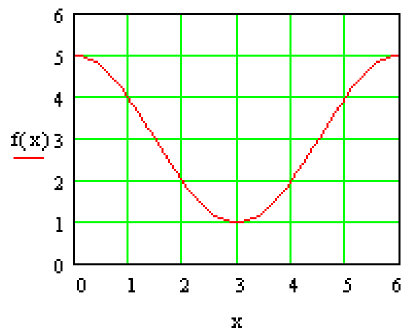
I have neither given nor received aid in the completion of this examination.

Signature: ______________________________

Please do not write below this line.

Page 1 ________	Page 5 ________	Page 9 ________
Page 2 ________	Page 6 ________	Page 10 ________
Page 3 ________	Page 7 ________	
Page 4 ________	Page 8 ________	TOTAL:

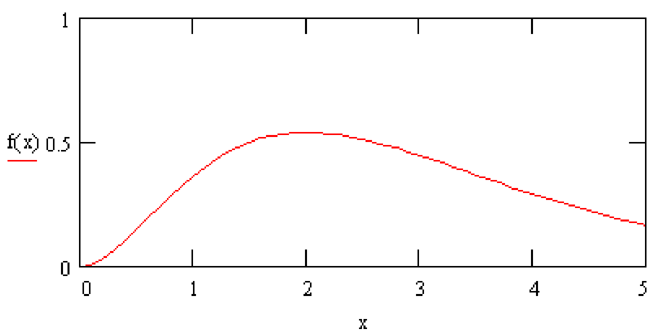
1. (6 points) Find a possible formula for the graph of the trigonometric function given below:



$f(x) =$ ______________________

2. (8points) On the graph below, illustrate what is meant by

$\sum_{k=1}^{6} f(x_k)\Delta x$ where $\Delta x = \frac{1}{2}$ and $x_k = 1 + k\Delta x$.



3. (4 points each) Compute the following integrals exactly (do not use a calculator). As is required for all questions on this exam, you must show your work so that your reasoning is clear.

(a) $\int_1^a \frac{1}{x}\,dx$ [Assume $a > 1$.]

(b) $\int_0^1 \sqrt{1 - x^2}\,dx$

4. (12 points) Suppose we roll two six-sided dice. Let X be the random variable that represents the absolute value of the difference of the values that come up.

(a) What values could X possibly assume?

(b) Graph the probability mass density function for X.

(c) What is the expected value of X?

5. (6 points) Suppose you collect the following data in a survey. (For example, the following numbers could be scores of 10 randomly selected people on some test.)

4.2, 4.4, 6.1, 6.1, 7.5, 7.8, 8.1, 8.8, 9.1, 9.4

(a) If F is the cumulative distribution function, find $F(7)$.

(b) Let f be the probability density function. Estimate $f(7)$. Explain how you got your answer.

6. (5 points) Consider $p(x)=\begin{cases} cx-x^2 \text{ if } 0\le x\le c \\ 0 \text{ otherwise} \end{cases}$. Find c so that $p(x)$ is a probability density function.

7. (5 points) Let $f(x)=\begin{cases} \frac{c}{x} \text{ if } x>1 \\ 0 \text{ otherwise} \end{cases}$. Could $f(x)$ be a probability density function for an appropriate c? Why or why not?

8. (10 points) The following table gives the amount of sunlight in Durham NC on the day given. Let $t=0$ correspond to March 21. Find a sinusoidal function that would give the amount of sunlight on any given day. Explain your answer in a short paragraph.

March 21	12.1 hours
June 21	14.5 hours
September 21	12.1 hours
December 21	9.7 hours

9. (4 points) State the definition of $\int_a^b f(x)dx$, the definite integral from a to b.

10. (8 points) Circle all of the following that are solutions to the differential equation $\frac{dy}{dx} = y - 2x$.

(a) $y = 2x + 2 + e^x$

(b) $y = 2x + 2 - e^x$

(c) $y = e^x - x^2$

(d) $y = e^x - x^2 + 1$

11. (6 points)

(a) If $y = arctan(x)$, then $x =$ ______________

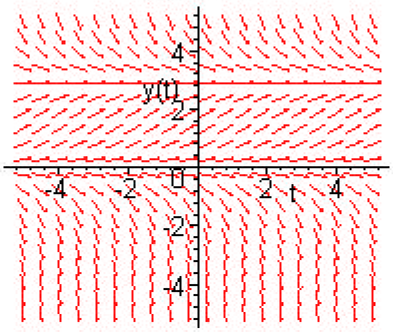
(b) Use your answer in (a) above to show carefully why $\frac{d}{dx}(arctan(x)) = \frac{1}{1+x^2}$.

12. (12 points) Match the slope fields below with the given differential equations:

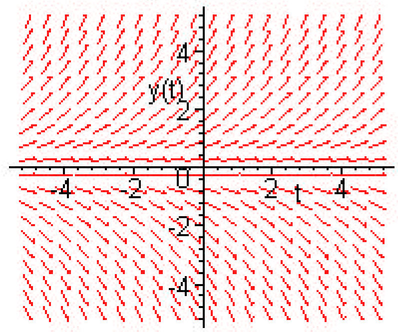
$\frac{dy}{dt} = \frac{1}{2}y$ ________ $\qquad$ $\frac{dy}{dt} = \frac{1}{2}t$ ________

$\frac{dy}{dt} = \frac{3}{10}y(3-y)$ ________ $\qquad$ $\frac{dy}{dt} = \frac{3}{10}t(3-t)$ ________

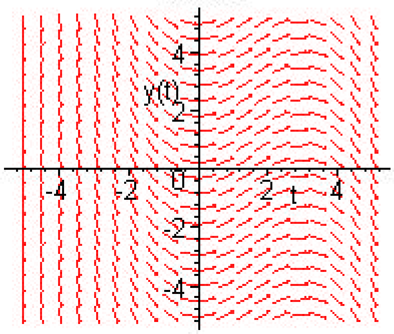
Graph A



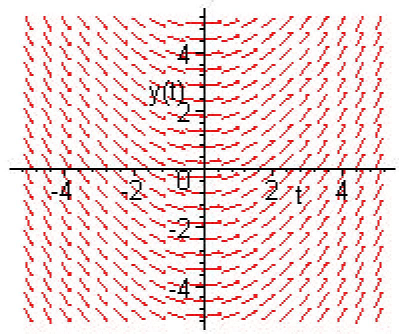
Graph B



Graph C



Graph D



13. (8 points) Below are 4 differential equations that are models of population growth discussed in class. In each case, $k > 0$ and $M > 0$. Match each differential equation with one of the situations below that could be modeled most appropriately by that differential equation.

$\frac{dP}{dt} = kP$	a	b	c	d
$\frac{dP}{dt} = k(P-M)$	a	b	c	d
$\frac{dP}{dt} = kP(M-P)$	a	b	c	d
$\frac{dP}{dt} = kP+M$	a	b	c	d

(a) unrestricted (or natural) population growth

(b) the spread of a rumor

(c) natural population growth with a constant rate of immigration

(d) Newton's Law of Cooling

14. (10 points) A car going 80 feet per second (about 55 mph) brakes to a stop in 5 seconds. Assuming that the deceleration was constant, how far did it travel in those 5 seconds?

15. (4 points) Let $F(x) = \int_3^x f(t)\,dt$, where $f(t) > 0$. Assume that h is a small, positive number. Circle every expression below which is equivalent to $\frac{F(x+h)-F(x)}{h}$.

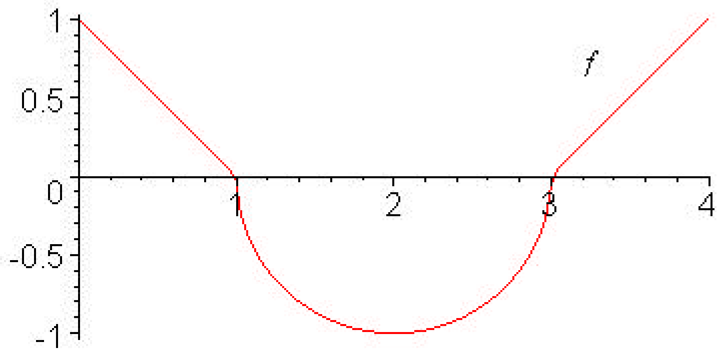
- $f(x+h)$
- $F'(x)$
- $F''(x)$
- $f(x)$
- $f'(x)$
- $F(3)$
- an approximation of $F'(x)$
- area under F over $[3,x]$
- an approximation of $f(x)$
- the average of F over $[x,x+h]$
- an approximation of $F'(x)$
- area under f over $[x,x+h]$
- the average of f over $[x,x+h]$
- area under F over $[x,x+h]$

16. (4 points each) The following statements may be correct or may contain one or more errors. State whether or not it is correct and explain why.

(a) $\int_0^2 x(x^2+1)^3\,dx = \int_0^2 \frac{1}{2}u^3\,du = \frac{1}{8}u^4 \Big|_0^2 = 2$

(b) $\int_0^{\frac{\pi}{2}} x\cos(x)dx = x\sin(x) - \int_0^{\frac{\pi}{2}} \sin(x)dx = x\sin(x) + \cos(x)\Big|_0^{\frac{\pi}{2}} = x\sin(x) - 1$

17. (20 points) Consider the function, f, defined by the graph below(the parts that appear curved are parts of circles and the parts that appear straight are straight).
Let $F(x) = \int_0^x f(t)\,dt$ for x in the interval $[0,4]$.



(a) Find $F(0), F(1), F(2), F(3)$, and $F(4)$, and write your answer in the table.

x	$F(x)$
0	
1	
2	
3	
4	

(b) Find the average value of f on the interval $[0,4]$.

(c) Find where $F'(x) = 0$ on the interval $[0,4]$ and label each of the point(s) as either a local maximum or a local minimum.

18. (24 points) Solve the following initial value problems exactly, expressing y explicitly as a function of x.

(a) $\frac{dy}{dx} = 3 - 1.5y, \quad y = 1$ at $x = 0$

(b) $\frac{dy}{dx} = 3 - 1.5\sqrt{x}, \quad y = 1$ at $x = 0$

(c) $\frac{dy}{dx} = \frac{x}{y}, \quad y = 1$ at $x = 0$

19. (18 points) Write an integral or a differential equation that models the following situations.

(a) An ingot of iron ore at 1200°F is plunged into a water bath whose temperature is kept at 70°F. The temperature of the ingot decreases at a rate proportional to the difference between and the temperature of the ingot and the surrounding temperature. If the proportionality constant is 3, find, but DO NOT EVALUATE, an expression for the temperature as a function of time.

(b) Suppose that the rate at which the world's oil is being consumed is continuously increasing and can be modeled with the function $r(t) = e^{kt} + e^{-kt}$, where r is measured in barrels per year and t is measured in years since January 1, 1990. Find, but DO NOT EVALUATE, an expression for how much oil was used between January 1, 2000 and January 1, 2002.

(c) Dead leaves accumulate on the ground in a forest at a rate of 3 grams per square centimeter per year. At the same time, these leaves decompose at a continuous rate of 75% per year (i.e. the leaves decompose at a rate proportional to the amount present with proportionality constant 0.75). If there are 10 grams per square centimeter of leaves on the ground now, find, but DO NOT EVALUATE, an expression for how much will be there in 5 years?

20. (18 points) Suppose that your company will sell about 300,000 computer chips over the next year. Your statisticians have determined that the function, $g(t) = 1 - e^{-.008t}$ represents the fraction of the total number of computer chips that have failed as of time t (in months).

(a) Is $g(t)$ a probability density function or a cumulative distribution function? Explain.

(b) What is the probability that a computer chip will last more than a year? How many chips would you expect to last more than one year?

(c) What is the average (or mean) lifetime of a computer chip?

(d) What is the median lifetime of the computer chips?