Math 32L Lab Quiz #3 (Limited Immunity)

Blake, Spring 2002

Name: ________________________

1. (21 points) In the S-I-R-S model for the spread of an infectious disease, we constructed the following differential equations:

$$\frac{ds}{dt} = -\beta s(t)i(t) + \mu(1 - s(t) - i(t))$$

$$\frac{di}{dt} = \beta s(t)i(t) - \lambda i(t)$$

(a) Which of the constants is determined by the rate at which people recover from the disease? [Circle one answer.]

β $\qquad$ λ $\qquad$ μ $\qquad$ 1 $\qquad$ none of these

(b) Which of the constants could be affected by a public awareness campaign? [Circle one answer.]

β $\qquad$ λ $\qquad$ μ $\qquad$ 1 $\qquad$ none of these

(c) Circle all the nullclines for these differential equations.

$s = 0$ $\qquad$ $i = 0$ $\qquad$ $i = \frac{\mu(1-s)}{\beta s+\mu}$ $\qquad$ $s = \frac{\lambda}{\beta}$ $\qquad$ $i = \frac{\beta}{\lambda}$ $\qquad$ $s = i$ $\qquad$ $s = 1 - i$

(d) In the setup for the "Limited Immunity" model, we introduced a term, μR, which did not exist in the original S-I-R model. Why did we introduce this term? [Check one answer.]

___ We assumed that some people will never recover from this disease.

___ We assumed that some people who are susceptible will move directly into the "recovered" group, thereby bypassing the "infected" group.

___ Math problems are easier to work with if they contain at least three Greek letters.

___ We assumed that a fraction of the "recovered" group would become susceptible again.

(e) Suppose that the equilibrium point in the (s,i) phase plane is (0.6, 0.1), and that we have modeled a disease for a city of 10 million people. What is the implication of this equilibrium? [Use specific numbers, and state your answer clearly with complete sentences.]

2. (24 points) Above each of the phase planes below write the letter corresponding to the system of differential equations represented by the phase plane. On each phase plane pictured, indicate all the null clines (please make them **dark** enough to see easily), and in each region draw directional arrows to indicate the direction of the flow.

System A:
$\frac{dx}{dt} = 0.02y$
$\frac{dy}{dt} = 0.01x$

System B:
$\frac{dx}{dt} = -0.4yx + 0.09(1 - y - x)$
$\frac{dy}{dt} = 0.4yx - 0.2y$

System C:
$\frac{dx}{dt} = -x + xy$
$\frac{dy}{dt} = y - yx$

System D:
$\frac{dx}{dt} = x(2 - x - y)$
$\frac{dy}{dt} = y(1 - x - y)$

System E:
$\frac{dx}{dt} = 1 - xy$
$\frac{dy}{dt} = y(1 - y)$

System F:
$\frac{dx}{dt} = 1 - x - y + 0.8xy$
$\frac{dy}{dt} = 0.4yx - 0.24y$

System G:
$\frac{dx}{dt} = 0.8x - xy$
$\frac{dy}{dt} = -y + 0.8yx$

System H:
$\frac{dx}{dt} = xy - x$
$\frac{dy}{dt} = xy - y$

[Pictures of phase planes are missing.]