Present Value and Future Value Report

Section: ______ Team number: _____

Team members: ____________________________ ____________________________

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Part 1: Future Value and Present Value of a Sequence of Payments

Suppose Publishers' Clear Glass House is advertising its “$1 million” sweepstakes. But when you read the fine print, you discover that the $1 million will be paid to the winner in 40 annual payments of $25,000 each, starting on the day of the selection of the winner. Assume that you can deposit money in an account which pays 6% interest, compounded twice a year.

1. What is the present value (PV) of the second $25,000 payment? The third $25,000 payment? The n^{th} $25,000 payment? Now compute the PV of the $1 million grand prize. Suppose you win and then your cousin, Gil Bates, offers you $400,000 for the grand prize. Would you take the offer? Why (not)?

2. While you're pondering the last offer from Gil, he returns to you with another offer. He says his company, Bikers' Loft, will pay you and your heirs $20,000 a year, starting now and continuing forever, if you'll turn the grand prize over to him. Compute the present value of this offer. Is this a better offer? Why (not)?

3. Suppose you discover a Savings and Loan that pays 6% interest, compounded continuously. How much does this change the PV of Gil's offer of $20,000 a year forever? Explain this change intuitively. Does this alter your response to Gil's latest offer?

Part 2: Future Value and Present Value of an Income Stream

Let $p(t)$ be the rate at which profit is "streaming" into Wal-More. We will compute the present value of this income stream for over a period of time. We assume that the current, continuously compounded interest rate is r.

1. First, we look at the next 10 years. We shall use the same strategy that we used in computing quantities such as volumes and arc length: divide this 10-year period into N subintervals, each of length Δt, with endpoints $\{t_0,\ t_1, \ldots,\ t_N\}$. We construct a Right Hand Sum to approximate the PV of this income stream:

(a) Explain why the expression $p(t_k)\,\Delta t$ would approximate the profit during the time from t_{k-1} to t_k.

(b) What expression would approximate the PV of the profit during the time from t_{k-1} to t_k ?

(c) Carefully describe the meaning of the expression $\sum_{k=1}^{N} e^{-rt_k}\,p(t_k)\,\Delta t$.

(d) As we let $N \to \infty$, the expression above takes on a value represented by a definite integral. Write down the integral, and explain what it means.

2. Suppose $p(t) = 50$ is a constant income stream (in units of \$1000 per year). Assume the current (continuously compounded) interest rate is 4%.

(a) Compute the PV of this income stream over the next 10 years.

(b) If we expect this income stream to last a long time, we could represent the situation mathematically by assuming it goes on forever. Write down an integral that would represent the PV of this stream under the assumption it will last forever. Does this integral converge? Explain intuitively how this result can make sense.

3. Suppose $p(t) = 1.5\,t$ (in units of \$1000 per year) models an increasing income stream, which we expect to last for a very long time.

(a) Assuming the current interest rate is 7% (compounded continuously), compute the PV of this income stream.

(b) Assuming the current interest rate is given by the constant r (compounded continuously), compute the PV of this income stream as a function of r. If this income stream were yours, under what conditions would you be willing to sell it for \$200,000?