Math 32L Quiz on Fourier Series

Blake, Spring 1999

40-point HW and Lab Quiz

Name: ________________________

Trigonometric Identities: Here are some nice trig identities.

$sin^2(\theta)+cos^2(\theta)=1$ $\quad$ $1+tan^2(\theta)=sec^2(\theta)$ $\quad$ $1+cot^2(\theta)=csc^2(\theta)$

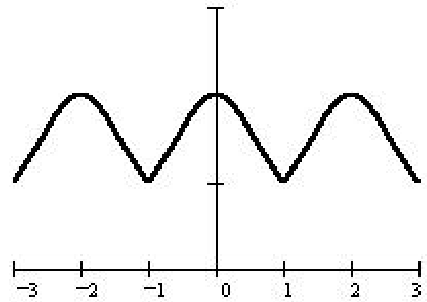
$sin(\alpha+\beta)=sin(\alpha)cos(\beta)+cos(\alpha)sin(\beta)$ $\quad$ $tan(\theta)=\frac{sin(\theta)}{cos(\theta)}$

$cos(\alpha+\beta)=cos(\alpha)cos(\beta)-sin(\alpha)sin(\beta)$ $\quad$ $\cos^2(\theta)=\frac{1}{2}(1+cos(2\theta))$

Nota Bene: In any part of this quiz, if you need to use the value of a well-known integral of trig functions, you may use that value without justification; e.g., the value of the integral $\int_0^{2\pi} sin(t)dt$ is known to be 0 and you may use that fact if you need it. If you use your calculator to compute any integrals, then make it clear what integral you are computing and indicate that you used your calculator to compute its value.

1. (12 points) Suppose that $f(x)$ is a periodic function which can be approximated well by a function of the form $a_0+a_1cos(x)+b_1sin(x)+a_2cos(2x)+b_2sin(2x)$. Fourier derived a formula for all of the a_k and b_k. Derive the specific formula for b_2. [Simply stating the formula is insufficient.] Your work must be neat, clear, and well-organized.

2. (13 points) Let $f(x)$ be the periodic function pictured at the right, one complete period of which is given by $\frac{1}{1+x^2}$ for $-1 \le x \le 1$.



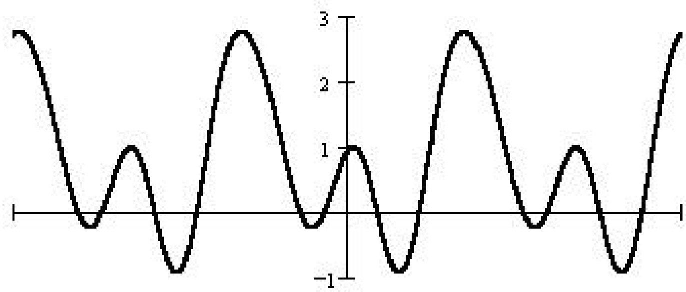
According to Fourier we can write this function in the form $a_0 + \sum_{k=1}^{\infty}(a_k cos(k\pi x) + b_k sin(k\pi x))$.

(a) Why does the number π appear in the arguments for the *sine* and *cosine* in this representation?

(b) Compute the values of a_1 and a_2 accurate to three decimal places.

3. (5 points) Let $f(x) = \frac{1}{2} sin(3x + \frac{\pi}{4})$. Find the Fourier series for this function without computing any integrals.

4. (10 points) The diagram below shows the graph of a function $h(t)$, which has a period of 2π. Some values of this function are shown in the table below the diagram. .



t	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$h(t)$	2.7	2.19	0.5	-0.166	0.9	0.21	-0.9	0.966	2.7

We can use the trapezoid rule to approximate the coefficient of cos($3t$) in the Fourier representation of h. Which expression below is the correct one for this approximation?

(a) $\frac{1}{4}\left(\frac{1}{2}(2.7)+2.19+.5-0.166+0.9+0.21-0.9+0.966+\frac{1}{2}(2.7)\right)$

(b) $\frac{1}{4}\left(\frac{1}{2}(-2.7)-2.19\frac{\sqrt{2}}{2}+0-0.166\frac{\sqrt{2}}{2}+0.9+0.21\frac{\sqrt{2}}{2}+0-0.966\frac{\sqrt{2}}{2}+\frac{1}{2}(-2.7)\right)$

(c) $\frac{1}{4}\left(\frac{1}{2}(0)-2.19\frac{\sqrt{2}}{2}+0.5+0.166\frac{\sqrt{2}}{2}+0+0.21\frac{\sqrt{2}}{2}+0.9+0.966\frac{\sqrt{2}}{2}+\frac{1}{2}(0)\right)$

(d) $\frac{1}{4}\left(\frac{1}{2}(-2.7)+2.19\frac{\sqrt{2}}{2}+0-0.166\frac{\sqrt{2}}{2}+0.9-0.21\frac{\sqrt{2}}{2}+0+0.966\frac{\sqrt{2}}{2}+\frac{1}{2}(-2.7)\right)$

(e) $\frac{1}{4}\left(\frac{1}{2}(-2.7\pi)-2.19\frac{3\pi}{4}-0.5(\frac{\pi}{2})+0.166\frac{\pi}{4}+0+0.21(\frac{\pi}{4})-0.9(\frac{\pi}{2})+0.966(\frac{3\pi}{4})+\frac{1}{2}(2.7\pi)\right)$