Math 31L Lab Quiz #2

Blake Fall 1999

Name: _____

1. (10 points) Use the differential equations and initial conditions below to find $y(t)$.

Be sure to show all your work. Unsupported answers will receive no credit.

$\frac{dy}{dt} = 2w,\ y(0)=4;$

$\frac{dw}{dt} = -2w,\ w(0)=5.$

(over)

2. (20 points) Consider the differential equations below, with the indicated initial conditions.

$$\frac{dy}{dt} = -5y + 7w \; ; \qquad \frac{dw}{dt} = 5y - 7w \; ; \qquad y(0) = 20 \text{ and } w(0) = 16.$$

(a) Explain clearlywhy $y(t)+w(t)$ must be constant. What is the constant?

(b) What makes the differential equation, $\frac{dy}{dt} = -5y + 7w$, more difficult to solve than the differential equations we have encountered before this lab? [Put a check beside the best answer.]

_____ There are two dependent variables.

_____ There is no t on the right-hand side.

_____ It's a second-order differential equation.

_____ $\frac{dy}{dt}$ is a linear function of y.

_____ Actually it's just as easy: you simply antidifferentiate term-by-term.

(c) To find $y(t)$ the first step would be to replace the differential equation, $\frac{dy}{dt} = -5y + 7w$, with one that's easy to solve. Do that now; i.e., produce an expression for $\frac{dy}{dt}$ which we can solve easily. Be sure to show all of your work. **Do not solve the differential equation that you give as the answer to this problem.**

(d) The function, $y(t)$, that is part of the solution to the system above is $y(t) = 21 - e^{-12t}$. Find $w(t)$.

(e) Show how you could find the equilibrium value of y <u>without</u> using the solution, $y(t)$. Be sure to justify your answers with clear work.

(f) Show how to use the solution to find the equilibrium value of y. Be sure to show your work clearly.