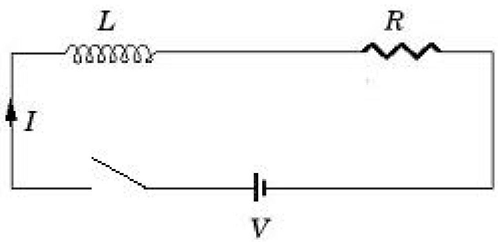
Mathematical Modeling with Differential Equations

<u>RL Circuit</u>



I is the amount of current flowing at time t.
V is the constant voltage from the battery.
R is the constant resistance of the resistor.
L is the constant inductance of the induction coil.

If we take time $t = 0$ to be the time at which the switch is closed, then $I = 0$ at time 0.

$$V = RI + L\frac{dI}{dt}$$

(a) Solve the DE for I in terms of t.

(b) What is the "steady state"? That is, what happens to I as $t \to \infty$? Do you recognize this answer–perhaps from work done in a prior physics lab?

(c) What effect does L have on this circuit? In particular, does L affect the steady state? Does L affect the rate at which the system moves to steady state?

A Summer Project

The cooling system in an old vehicle of mine held about 10 liters of coolant. One summer I flushed the cooling system by running water into a "tap–in" on the heater hose while the engine was running and simultaneously draining the resulting mixture of water and coolant from the bottom of the radiator. Water flowed in at the same rate the mixture flowed out— at about 2 liters per minute. The system was initially filled with pure antifreeze.

We will construct a differential equation to model the changing mixture in the vehicle's cooling system. Let $W(t)$ denote the amount of **water** in the system after t minutes.

(a) What is the value of $W(0)$?

(b) Because the engine was running, we can assume that the water and antifreeze in the system were thoroughly mixed. What fraction of the mixture flowing out was water? [Hint: your answer will involve $W(t)$.]

(c) Now complete the following statement of the initial value problem:

$$\frac{dW}{dt} = \text{(rate of input)} - \text{(rate of output)} = \qquad\qquad ; \; W(0) =$$

(d) Solve this IVP for $W(t)$.

(e) Compute $\lim_{t \to \infty} W(t)$. Is your answer consistent with your "common-sense" expectation? Why (not)?

(f) How long would it take for 90% of the mixture to be water?