

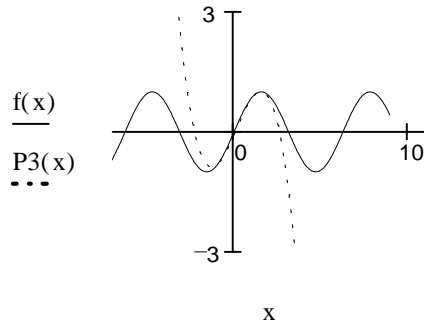
Taylor Approximations

Part 1: How high should the degree be?

$$f(x) := \sin(x) \quad x := -9, -8.99..9$$

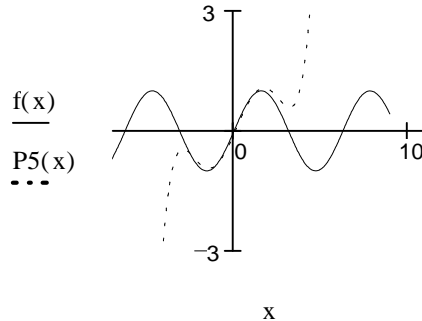
Third-order Taylor Polynomial

$$P_3(x) := \sum_{k=0}^1 \frac{(-1)^k \cdot x^{2 \cdot k + 1}}{(2 \cdot k + 1)!}$$



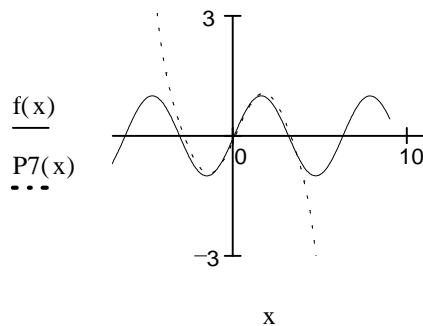
Fifth-order Taylor Polynomial

$$P_5(x) := \sum_{k=0}^2 \frac{(-1)^k \cdot x^{2 \cdot k + 1}}{(2 \cdot k + 1)!}$$



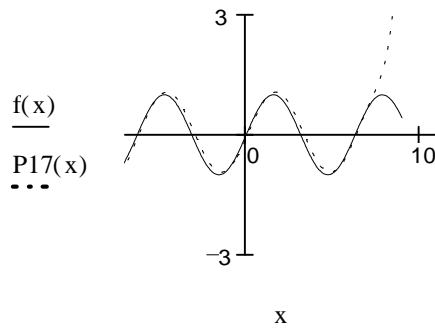
Seventh-order Taylor Polynomial

$$P_7(x) := \sum_{k=0}^3 \frac{(-1)^k \cdot x^{2 \cdot k + 1}}{(2 \cdot k + 1)!}$$



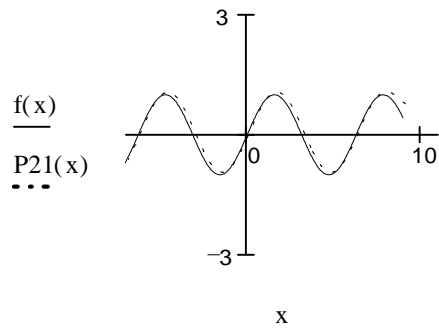
Seventeenth-order Taylor Polynomial

$$P_{17}(x) := \sum_{k=0}^8 \frac{(-1)^k \cdot x^{2 \cdot k + 1}}{(2 \cdot k + 1)!}$$



Twenty-first-order Taylor Polynomial

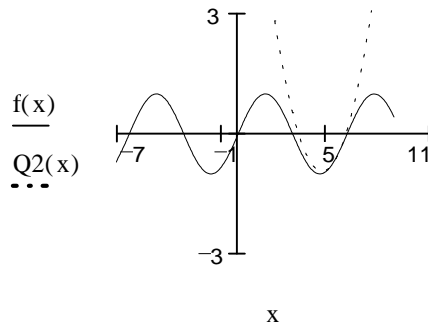
$$P_{21}(x) := \sum_{k=0}^{10} \frac{(-1)^k \cdot x^{2 \cdot k + 1}}{(2 \cdot k + 1)!}$$



Part 2: Changing the Base Point

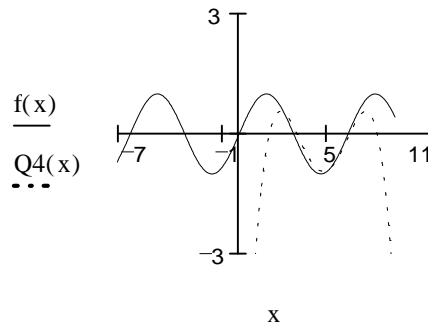
Second-order Taylor Polynomial

$$Q_2(x) := \sum_{k=0}^1 \frac{(-1)^{k+1} \cdot \left(x - 3 \cdot \frac{\pi}{2}\right)^{2 \cdot k}}{(2 \cdot k)!}$$



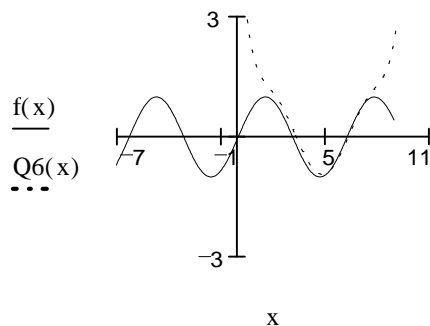
Fourth-order Taylor Polynomial

$$Q_4(x) := \sum_{k=0}^2 \frac{(-1)^{k+1} \cdot \left(x - 3 \cdot \frac{\pi}{2}\right)^{2 \cdot k}}{(2 \cdot k)!}$$



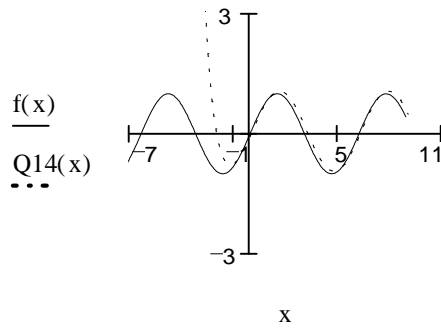
Sixth-order Taylor Polynomial

$$Q_6(x) := \sum_{k=0}^3 \frac{(-1)^{k+1} \cdot \left(x - 3 \cdot \frac{\pi}{2}\right)^{2 \cdot k}}{(2 \cdot k)!}$$



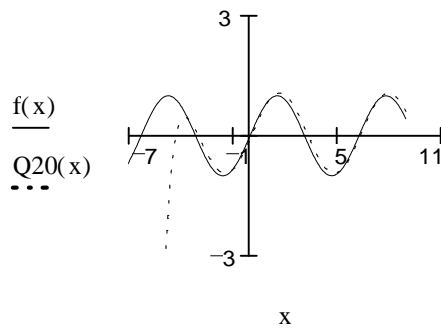
Fourteenth-order Taylor Polynomial

$$Q_{14}(x) := \sum_{k=0}^7 \frac{(-1)^{k+1} \cdot \left(x - 3 \cdot \frac{\pi}{2}\right)^{2 \cdot k}}{(2 \cdot k)!}$$



Twentieth-order Taylor Polynomial

$$Q_{20}(x) := \sum_{k=0}^{10} \frac{(-1)^{k+1} \cdot \left(x - 3 \cdot \frac{\pi}{2}\right)^{2 \cdot k}}{(2 \cdot k)!}$$

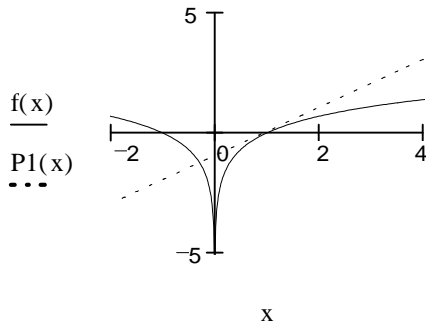


Part 3: Convergence Issues

$$f(x) := \ln(x) \quad x := -3, -2.99..5$$

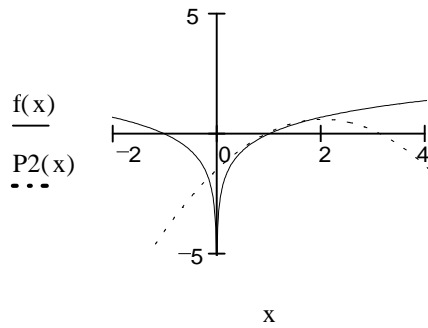
First-order Taylor Polynomial

$$P1(x) := \sum_{k=1}^1 \frac{(-1)^{k+1} \cdot (x-1)^k}{k}$$



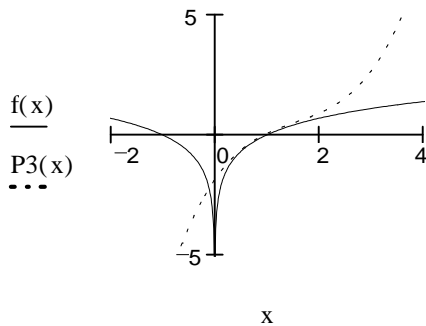
Second-order Taylor Polynomial

$$P2(x) := \sum_{k=1}^2 \frac{(-1)^{k+1} \cdot (x-1)^k}{k}$$



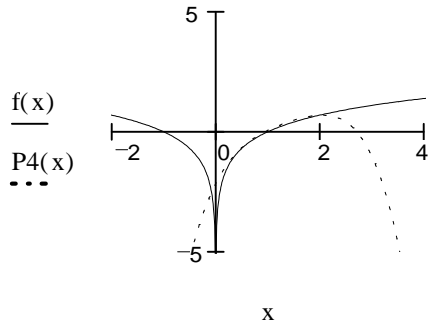
Third-order Taylor Polynomial

$$P3(x) := \sum_{k=1}^3 \frac{(-1)^{k+1} \cdot (x-1)^k}{k}$$



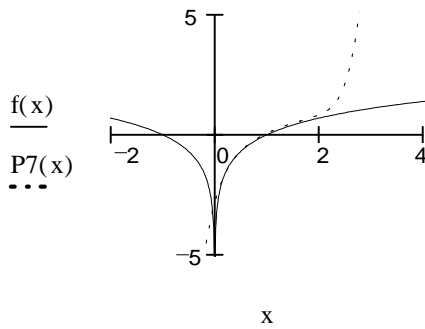
Fourth-order Taylor Polynomial

$$P_4(x) := \sum_{k=1}^4 \frac{(-1)^{k+1} \cdot (x-1)^k}{k}$$



Seventh-order Taylor Polynomial

$$P_7(x) := \sum_{k=1}^7 \frac{(-1)^{k+1} \cdot (x-1)^k}{k}$$



Twentieth-order Taylor Polynomial

$$P_{20}(x) := \sum_{k=1}^{20} \frac{(-1)^{k+1} \cdot (x-1)^k}{k}$$

