

## Computation of Selected Derivatives

1. Let  $f(x) = x^2$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\&= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} [2x + h] = 2x\end{aligned}$$

2. Let  $f(x) = x^3$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\&= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \rightarrow 0} [3x^2 + 3xh + h^2] = 3x^2\end{aligned}$$

3. Let  $f(x) = x^4$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h \rightarrow 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h} \\&= \lim_{h \rightarrow 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} = \lim_{h \rightarrow 0} [4x^3 + 6x^2h + 4xh^2 + h^3] = 4x^3\end{aligned}$$

4. Let  $f(x) = x^5$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h)^5 - x^5}{h} = \lim_{h \rightarrow 0} \frac{x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - x^5}{h} \\&= \lim_{h \rightarrow 0} \frac{5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5}{h} \\&= \lim_{h \rightarrow 0} [5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4] = 5x^4\end{aligned}$$