Oscillations

An Important Second-order Differential Equation

We saw that there were several ways we can approach the DE $\frac{d^2y}{dt^2} = -y$.

- We might guess an answer by thinking about derivatives we know.
- Realizing what this DE says about acceleration (i.e., it's in the opposite direction of the current position), we deduce it's describing an oscillation.
- Although we didn't pursue this approach in this course, one might use complex numbers; e.g., if $y = \mathrm{A}e^{it}$, then $\frac{dy}{dt} = iy$, and $\frac{d^2y}{dt^2} = -y$. The connection between this approach and oscillations is given by Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$.
- We could use series to solve this DE, just as we did a similar initial value problem in lab.

Here's the general result:

If $\frac{d^2y}{dt^2} = -\omega^2 y$ [where ω is a constant], *then the general solution is*

$$y(t) = A\cos(\omega t) + B\sin(\omega t)$$

Example: In an L-C electrical circuit, we get the following DE:

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = 0.$$

In this problem, L and C are constants (giving inductance and capacitance), and $Q(t)$ is the electrical charge at time t. We can put this DE in the form above, and then write the general solution.

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q.$$

The general solution is $Q(t) = A\cos\left(\frac{1}{\sqrt{LC}}\,t\right) + B\sin\left(\frac{1}{\sqrt{LC}}\,t\right)$.

If we had a value for $Q(0)$, we could determine the value of the constant A.
If we had a value for $Q'(0)$, we could determine the value of the constant B (by differentiating $Q(t)$ and plugging in 0 for t.)