Math 31L Homework Assignment
Linear Approximations

1. Let $f(x) = \frac{1}{x}$.
   (a) Compute the value of $f'(1)$ from the definition of the derivative of f at 1.
   (b) Find the tangent line approximation to $f(x)$ near $x = 1$.

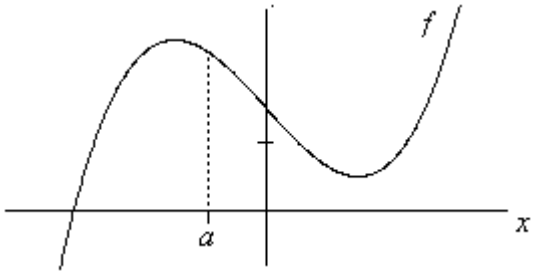
2. Let $f(x) = \sqrt{1+x}$.
   (a) Compute the value of $f'(0)$ from the definition of the derivative of f at 0. [Hint: use multiplication by the conjugate to rationalize the numerator of the difference quotient.]
   (b) Find the local linearization of $f(x)$ near $x = 0$.
   (c) Use your local linearization to approximate $\sqrt{1.01}$.
   (d) Would you expect your approximation in part (c) to be too large or too small? Explain your answer. Also, use your calculator to estimate the actual error and draw a picture to explain the situation.

3. (a) Show that $1 - x$ is the local linearization of $\frac{1}{1+x}$ near $x = 0$.
   (b) Use your answer in part (a) to show that, for values of x near 0, $\frac{1}{1+x^2} \approx 1 - x^2$

4. Suppose that f is concave up for all x, and Δx is a small positive number. Which is larger, $f(1 + \Delta x)$, or $f(1) + f'(1)\Delta x$? Explain your answer with the help of a picture.

5. Suppose f is concave down for all x, and Δx is a small positive number. In each of the following pairs, which number is the larger? Give a reason for your answer.
   (a) $f'(5)$ and $f'(6)$ (b) $f(5 + \Delta x)$ and $f(5) + f'(5)\Delta x$

6. Suppose that f is the curve pictured at the right. The number a is constant. Draw the graph of the function $L(x) = f(a) + f'(a)(x - a)$ on the same diagram.



7. (a) Why would you expect the following equation to have a solution near 0?
$$x + sin(x) - .01 = 0$$
   (b) Replace $sin(x)$ with its local linearization at (0,0), and then use this new equation to approximate a solution to the first equation.
   (c) Now use your calculator to find a solution to of the original equation accurate to 8 decimal places. How much error was in your approximation done in part (b)?

Several of these problems are taken from *Calculus*, by Hughes-Hallett, et al.