Related Rates Homework

1. A Japanese beetle infestation is spreading from the center of a small town. Since beetles fly in all directions, we assume the region they cover is circular. Suppose the radius of this circular region is increasing at a rate of 1.5 miles per year. Determine the rate of change of the area of infestation when the radius is 4 miles.

2. A manufacturer of tennis balls decides to increase production by 30 cans each day for the foreseeable future. The manufacturer has determined that the total revenue from the sale of x cans in a day is approximately

$$R(x) = 2.14x \; - \; .0001x^2 \text{ dollars.}$$

Determine the rate of change of revenue with respect to time when the daily production level is 1500 cans. (Assume that all cans are sold.)

3. An airplane is flying at a height of 5 miles and is traveling at a speed of 400 *mph* toward Denver. When its horizontal distance from Denver is 13 miles, how fast is the line–of–sight distance between the plane and Denver changing?

4. Suppose that a man 6 feet tall is walking at a speed of 8 *ft/sec* away from a street light, which is atop an 18-foot pole. How fast is the tip of his shadow moving along the ground?

5. A 13-foot ladder resting on horizontal ground is leaning against a vertical wall when its base starts to slide away from the wall. At the time the base is 12 feet from the wall, the base is moving at the rate of 10 feet per second. How fast is the top of the ladder sliding down the wall then? How fast is the area of the triangle formed by the ladder, wall, ground changing?

6. Suppose that a gas balloon is being filled at the rate of 100π cubic centimeters of helium per second. At what rate is the radius of the balloon increasing when the radius is 10 *cm*? At what rate is the radius of the balloon increasing when the radius is 100 *cm*? Compare your two answers, and explain why the *relative* sizes of the answers are plausible.

7. Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate.

Answers

1. It is increasing at $12\pi\ mi^2/yr$.

2. It is increasing at \$55.20/day.

3. It is decreasing at 373.34 mph.

4. It is moving at 12 ft/sec away from the pole.

5. The top of the ladder is falling at 24 ft/sec, and the area is decreasing at 119 ft^2/sec.

6. When the radius is 10 cm, the radius is increasing at .25 cm/sec. When the radius is 100 cm, the radius is increasing at .0025 cm/sec. As the balloon becomes larger and larger, the constant increase in volume should have a smaller and smaller effect on the total volume and hence on the radius. On the other hand, initially, when the radius is very small, we would expect the radius to begin increasing at a very rapid rate. This phenomenon is reflected by the appearance of r^2 in the denominator of the expression for $\frac{dr}{dt}$.