An Interactive Online Calculus Text

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Abstract

We describe the development and use of an online textbook for first-year calculus, the second edition of a book that first appeared in print. The principles on which the book is based are the same as those the first two authors developed in Project CALC more than a decade ago, but the online version has many interactive features that could not be in a print-based text. We expect the book to be published by the Mathematical Association of America, but for the time being (while development continues), it is freely available to all at http://www.math.duke.edu/education/calculustext. We are eager to have teachers and students use any or all of it and provide feedback.

Keywords: Calculus, single-variable, differential, integral, online, interactive, computer algebra system
AMS Subject Classification: 97-01, 97D30, 97D40, 97D80, 97U20, 97U70

1. Background: Project CALC

The textbook we describe here is the second edition of *Calculus: Modeling and Application*, a book that was first developed as part of Project CALC: Calculus As a Laboratory Course. This project was funded by the National Science Foundation (NSF) from 1988 to 1995, and the first edition textbook and lab materials were published in 1996 by Houghton Mifflin, shortly after acquiring the intended publisher, D. C. Heath. Some online lab modules were later developed as part of the Connected Curriculum Project, also funded by NSF from 1993 to 2001. These print and online materials are still in use at some colleges.

Project CALC had (and still has) the following characteristics:

- hands-on activities
- discovery learning
- real-world applications
- writing and revision of writing
- high expectations of students
- teamwork
- intelligent use of available tools
- emphasis on students checking their own work

These features were selected on the basis of research on what educational strategies lead to durable and transferable learning, as well as on modeling what students could be expected to encounter once they leave the academic world.

Early on we established goals for what we expected Project CALC students to learn and do, while in each semester of the course, as well as after the course was over. For example, here are some of our in-course goals:

Students in the Project CALC sequence should be able to

- understand concepts rather than merely mimic techniques;
- demonstrate understanding by explaining in written or oral form the meanings and important applications of concepts;
- construct and analyze mathematical models of real-world phenomena, including both discrete and continuous models;
- distinguish between discrete and continuous models and make judgments about the appropriateness of the choice for a given problem;
- understand the relationship between a process and the corresponding inverse process;
- select between formal and approximate methods for solution of a problem and make judgments about the appropriateness of the choice;
- select the proper tool or tools for the task at hand.

And at the end of the course:

Students who complete the Project CALC sequence should be able to

- use mathematics to structure their understanding of and investigate questions in the world around them,
- use calculus to formulate problems, to solve problems, and to communicate their solutions of problems to others,
- use technology as an integral part of this process of formulation, solution, and communication,
- work and learn cooperatively.

In 1991, Project CALC won the EDUCOM Higher Education Software Award as *Best Mathematics Curriculum Innovation*, and in 1993 it was cited by Project Kaleidoscope as *A Program that Works*.

2. The Second Edition

Our desires for the second edition are to

a) Make the text flexible, hyperlinked, interactive, richly illustrated, and available at low cost, and
b) Demonstrate the feasibility of an online textbook.

We are supported in this effort by an NSF grant (NSF-0231083) to the Mathematical Association of America (MAA) for the MathDL Books Online Project, which is also supporting development of a Mathematical Modeling text by Frank Wattenberg and his colleagues at the US Military Academy, as well as two demonstration chapters of David Bressoud's *A Radical Approach to Real Analysis*.

The first step in our process has been to redesign and redevelop the textbook entirely online, and that step is essentially complete. Some of the issues we encountered in this phase were page design, navigation, directory and file structure, sources for illustrations, nature and implementation of the built-in interactivity, technical requirements for the user, and effective presentation of mathematics online. Here are brief comments on how we dealt with those issues.

The basic page design and a cascading style sheet (CSS) for implementing it were provided for us by an in-house designer at MAA. We constructed our own hierarchical directory structure, based on chapters and sections of the book, with consistent naming of file types across chapters, and relative internal links, so that the entire structure could be moved without breaking any links.

Our navigation is based on a pop-up Table of Contents window for each chapter that remains open beside or overlapping the main text window. Supplementary notes, comments, and checkpoints for students also open pop-up windows that can be closed when they are no longer needed. Each main page has forward and back buttons, as well as a link to Contents page.

Our illustrations are of two kinds – mathematical graphs or diagrams and photographs or other illustrations. For the first kind, we construct a mathematically correct graph in a computer algebra system (usually Maple®) and then edit it in a graphic tool (usually Paint Shop Pro®). For the second, we find public domain or otherwise free photos (e.g., from US government sites or one of several free stock photo sites) or we take our own digital pictures, and then we crop, resize, etc. (again in Paint Shop Pro®).

We have a variety of interactive features, ranging from low to high on a technology scale. For example, we have a built-in pop-up numeric calculator, as well as files from which a student can print simple graph layouts, detailed graph paper, or slope fields for sketching solutions of differential equations. We have some embedded applets for carrying out certain experiments, and we have many prepared computer algebra (CAS) files in Maple® and Mathcad® that each get students started on an assigned task, but that will not completely solve the problem without student thought and inputs.

Our main text pages are constructed in XHTML, with most of the formulas presented in MathML, using code constructed in WebEQ®. We also use ASCIIMathML (a free product available under the GNU General Public License) for some of our formulas, including all the ones in pop-up pages, which are ordinary HTML.

To support our use of MathML across platforms, we require that the user have or install the Firefox browser and Mozilla's MathML fonts. To support ASCIIMathML and our control code for pop-ups and other interactions, we require that the user enable javascript. And to support our CAS activities, we require (at present) either Maple® (version 9.5 or later) or Mathcad® (version 13 or later). Our embedded applets require the Flash® player, which is included with all modern browsers, but it's also available as a free download.

3. What's in the Book?

In this section we present a brief summary of each chapter, including some of our reasons for structuring the book as we have.

Chapter 1: Relationships. The central question of the introductory chapter – which contains no calculus – is "What is a function?". Our objective is to separate this concept from other relationships between varying quantities and especially to separate it from "formula". Our purpose is to replace some of the inappropriate ideas students typically bring from secondary mathematics with a healthy regard for the mathematical concept that will be the foundation for the rest of the course. We also take up the algebra of functions and pose some problems for which the solutions are functions or classes of functions (e.g., symmetry, additivity).

Chapter 2: Models of Growth: Rates of Change. Here we establish some basic reasons for studying calculus, especially to be to able solve differential equations. Our primary example is the natural population growth equation, the simplest ODE to solve, and an immediate reason for moving beyond polynomials. We introduce difference quotients, derivatives, slope fields, initial value problems, solutions (which are, of course, functions or families of functions), exponential and logarithmic functions, and logarithmic plotting. The primary tool for understanding the derivative is zooming in on locally linear functions, and the primary formula is "slope equals rise over run."

Chapter 3: Initial Value Problems. This short chapter builds on Chapter 2, introducing Newton's Law of Cooling (exponential decay) to solve a murder mystery, then studying falling objects without air resistance (polynomial solutions).

Chapter 4: Differential Calculus and its Uses. This is the heart of the first-semester course, consolidating what has been learned about derivatives to take up optimization, concavity, Newton's Method (as an exercise in local linearity), and the basic formulas for differentiation. The product rule is introduced to study the growth rate of energy consumption, the chain rule to study reflection and refraction, and implicit differentiation to calculate derivatives of the logarithmic functions and of general powers. Zooming in is related to differentials and Leibniz notation.

Chapter 5: Modeling with Differential Equations. Here we return to falling bodies (e.g., raindrops, skydivers) and introduce air resistance proportional to the velocity or its square. The latter requires (for now) numerical solutions, and we take up Euler's Method as another "slope equals rise over run" application. We introduce periodic motion (with second-order ODE's, harking back to Chapter 3, where we derived position from constant acceleration), along with the basic trigonometric functions and their derivatives. This chapter concludes the first semester, and at the end of the chapter we summarize the derivative calculations.

Chapter 6: Antidifferentiation. At the start of the second semester, we turn our derivative summary inside out and catalog the functions for which we can now find antiderivatives – a necessary step if we're going to solve differential equations. We expand our tool kit with the simplest case of partial fractions to solve the Verhulst (logistic) model of population growth and explore how Verhulst, writing in 1840, could predict the US population in 1940.

Chapter 7: The Fundamental Theorem of Calculus. The big moment everyone has been waiting for – we introduce the integral as an averaging process, e.g., finding average temperature over a day or a year, and then relate that to area under a curve. We approach the FTC by exploring the linkage between speedometer and odometer, and then we "derive" the theorem by solving a differential equation – given the derivative, what's the function? – a question for which we already know one kind of answer. The partial sums of the left-hand rectangular approximations to area are, in fact, the Euler approximations to the solution of the differential equation, and this establishes the connection between antidifferentiation and area. Given this connection, it makes sense to introduce the indefinite integral as a notation for antidifferentiation.

Chapter 8: Integral Calculus and its Uses. This is the second-semester analog of Chapter 4. We start with a problem of fundamental physical importance, moments and centers of mass, to reinforce the idea of integration as averaging. We develop numerical methods through Simpson's Rule (as a weighted average of the trapezoidal and midpoint rules), so that no definite integral need remain unevaluated when one is working at a computer. Then we address the basic rules for integration by hand: algebraic and trigonometric substitutions and integration by parts. We close with an elementary look at Fourier analysis, using an electrocardiogram as an example.

Chapter 9: Probability and Integration. Our model problem in this chapter is reliability theory – how long do things last? The simplest model is the exponential distribution, which leads naturally into improper integrals. Now that students have experienced eight chapters of limiting behaviors, it is appropriate to introduce the standard notation for limits (but not the ε-δ definition, which belongs in a later course). We also take up other probability distributions (e.g., the normal) for which finding a mean or standard deviation may involve proper or improper integrals that can't be evaluated in closed form. This leads to defining some functions (e.g., the error function) by their integral representations.

Before we describe the last chapter, a brief polemic. It has become traditional in the US (and perhaps elsewhere) to end Calculus II with a chapter called "Sequences and Series", usually a compendium of everything we know short of a real analysis course, and invariably at a much higher level of sophistication than the rest of the course. The argument for doing this is that it's single-variable calculus, and the unstated argument is that it doesn't fit neatly anywhere else. There doesn't seem to be an argument that learning all possible convergence tests for series of constants is a skill needed by our future scientists and engineers. In our last chapter, we attempt to make the content flow naturally from what preceded it and to focus on skills that are or could be useful.

Chapter 10: Polynomial and Series Representations of Functions. As the title suggests, our emphasis is on representation of important functions, whether approximately by polynomials (perhaps very long polynomials) or by "infinitely long" polynomials. We start with the easy ones – exponential and trigonometric – and work up to the error function, using substitutions, differentiations, integrations. As a practical application, we note that it would take too long to evaluate the error function by integration, say, on a calculator or in a CAS, but it can be evaluated fast enough to graph it by using a relatively short polynomial. The primary tools for testing convergence are the alternating series test (AST) and the ratio test (RT) – and often they are the only tools needed. The first is geometrically obvious, and the second we obtain by comparing the tail of a series to that of a geometric series. Both come with error estimates. For power series, only the RT is needed unless there is a finite radius of convergence – and then the AST or comparison with, say, a harmonic series will usually do the trick.

4. Classroom Testing

Our textbook has been and is being classroom-tested at Hood College in Frederick, MD (USA) under the guidance of the third author, who is the lead teacher for calculus. Hood is a private, coeducational, liberal arts college with about 1200 undergraduate students, including a significant number of commuter students. The Project CALC materials have been used at Hood, under the leadership of department chair Betty Mayfield, almost from the beginning of their development, so it was natural for Hood to try the second edition.

In the Fall Term of 2006, the online text was used by 70 students in three sections with two teachers and four undergraduate teaching assistants (TA's). In Spring 2007, 45 of those students continued in Calculus II in two sections with two teachers and five TA's, and another 20 students started Calculus I with one instructor and one TA. In the current term (Fall 2007), Calculus I will again have three sections and Calculus II will have one (with both continuing and incoming students), and all sections will use the online text.

The challenges of using this book include getting students to accept an online text, learning how to use the text in class, convincing students to actually read the book, developing new versions of labs and projects, and coping with editing that was ongoing while the course was in progress. On the asset side of the ledger are the direct links to technology and to outside information, the checkpoints and activities with (slightly) "hidden" answers, and the opportunity to have a direct impact on the emerging edition.

At Duke our most recent teaching environment is the Interactive Computer Classroom [1], which functions as both laboratory and classroom as needed. This would be the ideal environment for use of an online text. The teaching environment at Hood is almost as good – each calculus section is scheduled simultaneously in a classroom and a lab that are next door to each other, allowing the instructor to move the class back and forth as necessary for the activity at hand. The classroom seats 24 at non-movable tables (plus an extra table for TA's), and the lab has 12 computers with two chairs each. Each room has an instructor's station with computer, document camera, VCR/DVD player, and ceiling-mounted projector. Each section is scheduled for three periods per week of an hour and 45 minutes.

A typical class day at Hood includes varied activities that might be any mix of discussion of the text (sometimes via lecture), working on a lab in pairs or on a project in somewhat larger groups, working on individual worksheets (possibly consulting with a neighbor), or writing a group report. Over the course of the year, the Hood classes covered most of the text but omitted periodic motion and circular functions in the first semester, as well as Fourier representations and some probability theory in the second semester. Parts of the omitted material were covered instead in labs, projects, or worksheets, either from the first edition (and eventually to be in the second edition) or of local design.

Here is our advice to instructors considering adoption of this online text:

- Be ready and willing to talk with your students about why your class is structured this way.
- Have the students read a bit about math education research, e.g., [2].
- Know what you want from your course: what's the focus?
- You can't be all things to all people.
- Exploration takes time, and there's no substitute for experience – the students'.
- Letting the class explore means you don't have complete control over what will happen next. Be flexible.
- Find ways to find out what your students really know.
- Work the projects ahead of time!
- As you teach, keep track of what you do when. (Some topics in the text are introduced at a surface level and revisited as tools become available.)
- If possible, find and hire TA's, especially for lab activities.

5. Previews of Coming Attractions

Over the course of the 2007-2008 academic year, we will complete the following additions and enhancements.

- Implementation of routine exercises in WeBWorK, a free online homework system that features randomly generated and individualized assignments, many ways to ask and answer questions, instructor-controlled numbers of repetitions and levels of hints, and automatic grading.
- An Instructor's Guide, with inputs from classroom teachers who have used and are using the text.
- Additional CAS options (e.g., Mathematica)

- Sections on the use and misuse of CAS integration tools (Ch. 8, to replace a first edition section on use of tables) and on convergence of a series to the right function (Ch. 10).
- More projects drawn from our first edition text and lab manuals and from the Connected Curriculum Project (http://www.math.duke.edu/education/ccp/). So far, we have projects on the spread of AIDS (Ch. 2), on air traffic control (Ch. 4, develops ideas of related rates and optimization), and on the area of Crater Lake (Ch. 7).
- Enrichment material – more applications (e.g., the SIR model for spread of epidemics, pendulum motion, discrete logistic growth and chaos, the gamma distribution) and theory (e.g., the Mean Value Theorem, the logical underpinnings of the calculus)
- Search/Browse/Index capabilities.

References

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[2] Smith, D. A., 1996, Thinking about Learning, Learning about Thinking, in A. W. Roberts (Ed.), *Calculus: The Dynamics of Change*, MAA Notes No. 39 (Washington, DC: Mathematical Association of America). Available online at http://www.math.duke.edu/~das/essays/thinking/.