2019 DUKE MATH MEET TIEBREAKER ROUND

Problem 1 **Name** ________________

Time Limit: 10 minutes **School** ________________

Problem 1. Let $a(1), a(2), \ldots, a(n), \ldots$ be an increasing sequence of positive integers satisfying $a(a(n)) = 3n$ for every positive integer n. Compute $a(2019)$.

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Problem 2 Name ____________________

Time Limit: 10 minutes School ____________________

Problem 2. Consider the function $f(12x - 7) = 18x^3 - 5x + 1$. Then, $f(x)$ can be expressed as $f(x) = ax^3 + bx^2 + cx + d$, for some real numbers a, b, c and d. Find the value of $(a + c)(b + d)$.

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Problem 3 **Name** ____________________

Time Limit: 10 minutes **School** ____________________

Problem 3. Let a, b be real numbers such that $\sqrt{5+2\sqrt{6}} = \sqrt{a}+\sqrt{b}$. Find the largest value of the quantity

$$X = \frac{1}{a + \frac{1}{b + \frac{1}{a + \cdots}}}.$$