Duke Math Meet 2017
Tiebreak Round

1. Find the sum of all 3-digit positive integers $\overline{abc}$ that satisfy

$$\overline{abc} = \binom{n}{a} + \binom{n}{b} + \binom{n}{c}$$

for some $n \leq 10$.

2. Feng and Trung play a game. Feng chooses an integer p from 1 to 90, and Trung tries to guess it. In each round, Trung asks Feng two yes-or-no questions about p. Feng must answer one question truthfully and one question untruthfully. After 15 rounds, Trung concludes there are n possible values for p. What is the least possible value of n, assuming Feng chooses the best strategy to prevent Trung from guessing correctly?

3. A hypercube H_n is an n-dimensional analogue of a cube. Its vertices are all the points $(x_1, ..., x_n)$ that satisfy $x_i = 0$ or 1 for all $1 \leq i \leq n$ and its edges are all segments that connect two adjacent vertices. (Two vertices are adjacent if their coordinates differ at exactly one x_i. For example, (0,0,0,0) and (0,0,0,1) are adjacent on H_4.) Let $\phi(H_n)$ be the number of cubes formed by the edges and vertices of H_n. Find $\phi(H_4) + \phi(H_5)$.

4. Denote the legs of a right triangle as a and b, the radius of the circumscribed circle as R and the radius of the inscribed circle as r. Find $\frac{a+b}{R+r}$.