Duke Math Meet 2017
Tiebreak Round Solutions

Problem 1.

The only solutions are 134 ($n = 8$), 246 ($n = 9$) and 505 ($n = 10$). Thus, the answer is $\boxed{885}$

Problem 2.

Trung can ask a trivial first question (e.g. "Is p an integer?") in order to determine the truthful answer to the second question. Using this strategy, Trung can eliminate half of the possible integers each round. For example, the second question in the first round could be "Is p at least 45?" Thus, the least possible value of n is $\boxed{1}$.

Problem 3.

The 8 vertices of a cube in H_n will be identical in all but 3 coordinates. Thus, for H_4, exactly 1 coordinate will be the same for all 8 vertices. There are 4 possible positions for this coordinate, and two choices for the value of the coordinate (0 or 1), so H_4 contains 8 distinct cubes. By similar reasoning, H_5 contains $\binom{5}{2} \cdot 2 \cdot 2 = 40$ distinct cubes, so the answer is $\boxed{48}$.

Problem 4.

Draw the center of the incircle, and label it I. Then draw the radii tangent to a, b, c and label them P, Q, S respectively. Note that $AQ = AS = \frac{b+c-a}{2}$, $BS = BP = \frac{a+c-b}{2}$ and $CP = CQ = \frac{a+b-c}{2}$. Further note that $CPIQ$ must be a square, as $CP = CQ$, IQ is parallel to CP, and PI is parallel to CQ. Thus $r = \frac{a+b-c}{2}$ and since $R = \frac{c}{2}$, we have $r + R = \frac{a+b}{2}$. Thus the ratio is just $\boxed{2}$.