Duke Math Meet 2015
Relay Round Question 1

1A. A factory produces socks and hats, only in the colors of white and blue. The ratio of white hats to blue hats is 9:5. The ratio of white hats to white socks is 3:7. The factory produces 3 times more white hats than blue socks. If the total number of socks produced per day is 24, how many total hats are produced?

Duke Math Meet 2015
Relay Round Question 1

1A. A factory produces socks and hats, only in the colors of white and blue. The ratio of white hats to blue hats is 9:5. The ratio of white hats to white socks is 3:7. The factory produces 3 times more white hats than blue socks. If the total number of socks produced per day is 24, how many total hats are produced?

Duke Math Meet 2015
Relay Round Question 1

1B. There are n trees in a circle, with a squirrel on each tree. The squirrels like each other so they all want to gather on one tree. However, Alex is a very mean person and has decided that the if the squirrels don't abide by his rule, he will cut down all the trees and they will have no homes. Alex's rule is this: every minute, (exactly) two squirrels may jump to an adjacent tree. For how many values of n, $1 \leq n \leq 5 \cdot TNYWR$, can all squirrels gather on one tree?

Duke Math Meet 2015
Relay Round Question 1

1B. There are n trees in a circle, with a squirrel on each tree. The squirrels like each other so they all want to gather on one tree. However, Alex is a very mean person and has decided that the if the squirrels don't abide by his rule, he will cut down all the trees and they will have no homes. Alex's rule is this: every minute, (exactly) two squirrels may jump to an adjacent tree. For how many values of n, $1 \leq n \leq 5 \cdot TNYWR$, can all squirrels gather on one tree?

Duke Math Meet 2015
Relay Round Question 1

1C. There are $n = TNYWR$ players in a tennis tournament. The players are ranked from 1 to n. First game of the tournament is between player n and player $n-1$. The loser is eliminated and he is ranked on the n-th place in the tournament. The winner of this game plays player $n-2$. The winner of this game plays player $n-3$ and the loser gets eliminated and ranks on the $n-1$-th place. And so on. How many possible outcomes does this tournament have? An outcome is a ranking of the players.

Duke Math Meet 2015
Relay Round Question 1

1C. There are $n = TNYWR$ players in a tennis tournament. The players are ranked from 1 to n. First game of the tournament is between player n and player $n-1$. The loser is eliminated and he is ranked on the n-th place in the tournament. The winner of this game plays player $n-2$. The winner of this game plays player $n-3$ and the loser gets eliminated and ranks on the $n-1$-th place. And so on. How many possible outcomes does this tournament have? An outcome is a ranking of the players.

Duke Math Meet 2015
Relay Round Question 2

2A. Place three congruent circles inside an equilateral triangle $\triangle ABC$ of side length 4 such that each circle is tangent to the other two circles and tangent to exactly two sides of the triangle. Let r be the radius of one of these circles. Find $(r+1)^2$.

Duke Math Meet 2015
Relay Round Question 2

2A. Place three congruent circles inside an equilateral triangle $\triangle ABC$ of side length 4 such that each circle is tangent to the other two circles and tangent to exactly two sides of the triangle. Let r be the radius of one of these circles. Find $(r+1)^2$.

Duke Math Meet 2015
Relay Round Question 2

2B. Let $l = TNYWR$. In $\triangle ABC$, $AB = 3 \cdot l - 1$, $AC = BC + 2$, and $BC = 5 \cdot l$. There exists a circle with center O that is tangent to both $\overline{AB}$ and $\overline{BC}$ such that O lies on $\overline{AC}$. Find the radius of the circle. Write the radius as $\frac{a}{b}$ with a, b relatively prime integers and pass forward the number a.

Duke Math Meet 2015
Relay Round Question 2

2B. Let $l = TNYWR$. In $\triangle ABC$, $AB = 3 \cdot l - 1$, $AC = BC + 2$, and $BC = 5 \cdot l$. There exists a circle with center O that is tangent to both $\overline{AB}$ and $\overline{BC}$ such that O lies on $\overline{AC}$. Find the radius of the circle. Write the radius as $\frac{a}{b}$ with a, b relatively prime integers and pass forward the number a.

Duke Math Meet 2015
Relay Round Question 2

2C. Let $n = TNYWR$. Tony has a bag with $n/4$ red balls, $n/6$ blue balls, and 10 white balls. Tony blindfolds himself and chooses 3 balls at random. John tells him that he has at least one white ball. What is the probability he has exactly two white balls?

Duke Math Meet 2015
Relay Round Question 2

2C. Let $n = TNYWR$. Tony has a bag with $n/4$ red balls, $n/6$ blue balls, and 10 white balls. Tony blindfolds himself and chooses 3 balls at random. John tells him that he has at least one white ball. What is the probability he has exactly two white balls?