Duke Math Meet 2015
Individual Round

1. Find the minimum value of $x^4 + 2x^3 + 3x^2 + 2x + 2$, where x can be any real number.

2. A type of digit-lock has 5 digits, each digit chosen from $\{1, 2, 3, 4, 5\}$. How many different passwords are there that have an odd number of 1's?

3. Tony is a really good Ping Pong player, or at least that is what he claims. For him, ping pong balls are very important and he can feel very easily when a ping pong ball is good and when it is not. The Ping Pong club just ordered new balls. They usually order form either PPB company or MIO company. Tony knows that PPB balls have 80% chance to be good balls and MIO balls have 50% chance to be good balls. I know you are thinking why would anyone order MIO balls, but they are way cheaper than PPB balls. When the box full with balls arrives(huge number of balls), Tony tries the first ball in the box and realizes it is a good ball. Given that the Ping Pong club usually orders half of the time from PPB and half of the time from MIO, what is the probability that the second ball is a good ball?

4. What is the smallest positive integer that is one-ninth of its reverse?

5. When Michael wakes up in the morning he is usually late for class so he has to get dressed very quickly. He has to put on a short sleeved shirt, a sweater, pants, two socks and two shoes. People usually put the sweater on after they put the short sleeved shirt on, but Michael has a different style, so he can do it both ways. Given that he puts on a shoe on a foot after he put on a sock on that foot, in how many different orders can Michael get dressed?

6. The numbers $1, 2, ..., 2015$ are written on a blackboard. At each step we choose two numbers and replace them with their nonnegative difference. We stop when we have only one number. How many possibilities are there for this last number?

7. Let $A = (a_1 a_2 \ldots a_n)_{34}$ and $B = (b_1 b_2 \ldots b_n)_{34}$ be two numbers written in base 34. If the sum of the base-34 digits of A is congruent to 15 (mod 77) and the sum of the base 34 digits of B is congruent to 23 (mod 77). Then if $(a_1 b_1 a_2 b_2 \ldots a_n b_n)_{34} \equiv x \pmod{77}$ and $0 \le x \le 76$, what is x?(you can write x in base 10)

8. What is the sum of the medians of all nonempty subsets of $\{1, 2, \ldots, 9\}$?

9. Tony is moving on a straight line for 6 minutes–classic Tony. Several finitely many observers are watching him because, let's face it, you can't really trust Tony. In fact, they must watch him very closely– so closely that he must never remain unattended for any second. But since the observers are lazy, they only watch Tony uninterruptedly for exactly one minute, and during this minute, Tony covers exactly one meter. What is the sum of the minimal and maximal possible distance Tony can walk during the six minutes?

10. Find the number of nonnegative integer triplets a, b, c that satisfy
$$2^a 3^b + 9 = c^2.$$