# Duke Math Meet 2014
## Relay Round Question 1

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1B. Let $n = TNYWR$. Let $S = \{1, 2, ..., n\}$. We call $M$ a special subset of $S$ if it has four elements and it has the property that if $x \in M$ then at least one of $x - 1$ and $x + 1$ are in $M$. Find the number of special subsets of $S$.


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1C. Let $n = (TNYWR)^4$. An ant is located at the origin of the $xy$-plane at time 0. At time 1 it moves north to $(0, 1)$. The following 10 moves of the ant are at $(1, 1), (1, 0), (1, -1), (0, -1), (-1, -1), (-1, 0), (-1, 1), ($ Each move takes one second. The ant continues to move in the patter described above. Find the $x$-coordinate of the ant at time $n$.

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## Relay Round Question 2

2A. Suppose there are 7 kids sitting at a round table and each one of them has a card which has a number between 1 and 7 on it such that any two kids have a different number written down on their card. The professor notices that for each child the sum of the number written on his card and the numbers written on his two neighbors cards is greater than $n$. What is the largest $n$ for which this is possible?


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2B. Let $2d = TNYWR$. Suppose you have a circle inscribed in a square. such that there is space for a rectangle of size $d \times 2d$ into the corner of the square such that the rectangle touches the square. Calculate the size of the side of the square.


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2C. Let $k$ be the largest integer smaller than $TNYWR$. Let $n = 100k + 2$. Feng has $n$ pairs of socks in the washing machine. If the washing machine randomly eats socks and at the end of the washing cycle the machine returns $t$ socks, $0 \leq t \leq 2n$ with equal probability, then calculate the expected value of the number of complete pairs of socks returned.


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