

DUKE MATH MEET 2008: RELAY ROUND

In the Relay Round each team of six students will divide into two groups of three students each. There are two sub-rounds in the Relay Round, each group of three students will work together. In each of the two sub-rounds, the three students in a group will receive a different problem. The problems that the second and third students in each group will have the symbol “TNYWR” within the problem statement. This stands for “The Number You Will Receive.” This is because the first student in the group is supposed to write down the answer to his or her problem and pass that answer to the second student, and similarly the second student is to pass the answer to his or her problem to the third students. The second student will need the first student’s answer to completely solve the second question, and similarly the third student will need the second student’s answer to completely solve the third question. The first and second students can only pass numbers that are fully simplified to the second and third students; there are to be no stray symbols, algebraic expressions, or other marks. The one exception is that students can underline numbers to indicate, for example, whether their answer is a 6 or a 9.

The first and second students may pass as many answers as they wish; however, the third student can only submit answers after three minutes or six minutes, and only the third student’s answer will be graded. If a group submits an answer after six minutes then their answer after three minutes, if they gave one, will be discarded. If a group obtains the correct answer after three minutes, they will earn 4 points for their team. If a group obtains the correct answer after six minutes, they will earn 2 points for their team. Therefore, a group should not, for example, submit the same answer after six minutes that they did after three minutes. The moderator will give 15-second warnings before the three minute deadline and the six minute deadline.

FIRST RELAY ROUND

- 1A.** What is the largest prime number that divides $49^2 + 18^2$?
- 1B.** Let k be TNYWR and let $L = k - 20$. Jake has taken three tests so far in math class and his grades have been 87, 100, and 92. Suppose she gets a score of L on her fourth test. If m is the mean score of her first three tests and M is the mean score of her first four tests, what is $m - M$?
- 1C.** Let k be TNYWR and let $L = 15(|k| + 1)$. How many perfect squares are between 1 and L , inclusive?

SECOND RELAY ROUND

- 2A.** The problem, as originally stated was unclear as to whether the scale would say which pan was heavier.

You have seven gold coins, one of which is counterfeit and therefore weighs a slightly different amount than the other six coins. In order to find out which of the coins is the counterfeit, you have a two-pan balance scale, where you can load items in each of two pans and press a button on the scale and the scale will tell you if the items in the two pans weighs the same amount and if they do not, which pan is heavier. What is the minimum number of button-presses you need to completely determine which of the coins is counterfeit?

- 2B.** Let k be TNYWR and suppose that r , s , and t are the solutions to the polynomial $x^3 + 2x^2 + kx + 2 = 0$. Compute $r^2 + s^2 + t^2$.

- 2C.** Let k be TNYWR and let L be the largest integer less than or equal to $|k - 1|$. You have an unfair six-sided die with faces numbered from one to six with the property that for any two integers p and q between 1 and 6, inclusive, the ratio of the probability that you roll a p to the probability that you roll a q is exactly $\frac{q}{p}$. What is the probability that you roll an L ?