

# Duke Math Meet Relay Round

November 19th, 2005

1. A1 One side of a triangle has length 45. How many ordered pairs of positive integers  $(b, c)$  are there so that the triangle with side lengths  $b, c, 45$  is right and the side of length 45 is a leg of the right triangle?
2. Let  $n = TNYWR$ . Now calculate the area of a regular  $n - 3$ -gon with sidelength 1 .
3. Present  $TNYWR$  in the form  $\frac{12}{a-\sqrt{b}}$ , where  $a$  and  $b$  are positive integers. Find the number of positive integers between 1 and  $25(b - a)$  that are relatively prime with 2005.
4. Find the last non-zero digit of  $25!$ .
5. Let  $n = TNYWR - 1$ . If  $f(x)$  is a polynomial of degree  $n$  with  $f(i) = i(i + 1)(i + 2)$  for  $n = 0, 1, \dots, n$ , find  $f(n + 1)$ .
6. Let  $n = TNYWR$ . You and your friends set up a marble race down a  $d$  meter long hill. Assume you have  $n$  marbles with constant speeds  $\{1, 2, 3, \dots, n\}$  in meters per second. Calculate the average time delay between consecutive marbles reaching the bottom of the hill.