On some invariants of singularities

I will survey different approaches to an invariant of singularities
called *log canonical threshold*. It appears in many situations related
to integrals of different flavors: usual integrals on $\mathbb{R}^n$ or $\mathbb{C}^n$, p-adic
integrals, or more recently, with motivic integrals. I will describe the
classical case having to do with complex powers in the work of Bern-
stein, Gelfand and Atiyah, emphasizing the connection with the Bern-
stein polynomial. I will then describe briefly the description via p-adic
integrals, due to Igusa, as well as a "motivic" version, involving spaces
of arcs.

In the last part of the talk I will discuss some conjectural connec-
tions between the log canonical threshold, and an invariant which ap-
pears in tight closure theory, the F-pure threshold. This time there
is no integration theory around, and the story has very much a pos-
itive characteristic flavor. Despite the fact that much of this part is
still conjectural, one can see a different connection with the Bernstein
polynomial.

The talk will be elementary and will not assume familiarity with any
of the concepts mentioned above. However, here are a few references:

(1) The classical part of the story, involving complex powers and
   p-adic zeta functions, is beautifully covered in:
      Jun-ichi Igusa, An Introduction to the theory of local zeta
   functions, AMS/IP Studies in Advanced Mathematics, 14. Amer-
   ican Mathematical Society, Providence, RI; International Press,
   Cambridge, MA, 2000.
(2) For the connection between spaces of arcs and the log canonical
   threshold (and other invariants, as well):
      Lawrence Ein, Rob Lazarsfeld, and Mircea Mustaţă, Contact
   loci in arcs spaces, math.AG/0303268.
(3) The speculative part concerning the relation between the log
   canonical threshold and the F-pure threshold is based on work
   in progress with Shunsuke Takagi and Kei-ichi Watanabe. For
   the definition and basic properties of the F-pure threshold, see
      Shunsuke Takagi and Kei-ichi Watanebe, On F-pure thresh-
   olds, math.AC/0312486.