MARKOV CHAINS ON PARTITIONS, PLANCHEREL MEASURES, AND POLYNUCLEAR GROWTH PROCESSES

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Consider the standard Poisson process in the first quadrant of the Euclidean plane. For any point (x,y) of this quadrant take the points of the Poisson point configuration sitting in the rectange with vertices $(0,0)$, $(x,0)$, (x,y), $(0,y)$. If we order these points by their x-coordinates then the order of the y-coordinates defines a permutation. This gives a 2-dimensional field of random permutations which is the main object of the talk.

Applying the Robinson-Schensted algorithm turns this field into a field of random partitions (or Young diagrams). At any given point (x,y) the partitions are distributed according to the so-called poissonized Plancherel measure which naturally arises in the representation theory of symmetric groups. It turns out that moving (x,y) along any south-east directed path in the quadrant is equivalent to running a Markov chain on partitions with transition probabilities generated by induction and restriction operations on the representations of the symmetric groups.

Another way to produce the same object is to consider the polynuclear growth model in one spatial dimension. Then the random interface is exactly the plot of the longest increasing subsequence of the permutation sitting at (x,y), or the size of the largest part of the corresponding partition.

The most interesting question is what happens when the point (x,y) is far from the origin. Remarkably, the rich algebraic structure of the model allows to compute explicitly the correlation functions along any south-east directed path and to control their asymptotics. The answer provides a striking connection with classical models of Random Matrix Theory.

References

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