HOMOLOGICAL MIRROR SYMMETRY FOR FANO SURFACES

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The phenomenon of mirror symmetry was first evidenced in the early 1990s as a remarkable correspondence between pairs of Calabi-Yau manifolds arising from a duality in string theory. Given a mirror pair (X, Y), the complex geometry of X is essentially equivalent to the symplectic geometry of Y, and vice-versa. The homological mirror symmetry conjecture, due to Kontsevich, treats mirror symmetry as an equivalence between two categories naturally attached to the mirror manifolds. Namely, *D-branes* (boundary conditions for open strings) are expected to be coherent sheaves in one of the two models of string theory, and Lagrangian submanifolds in the other model. Homological mirror symmetry hence predicts that the (bounded) derived category of coherent sheaves on X is equivalent to the (derived) *Fukaya category* of Y, and vice-versa.

We will focus on a different case, that of Fano manifolds ($c_1(TX) > 0$), which mirror symmetry puts in correspondence with *Landau-Ginzburg models*, i.e. pairs (Y, w) where Y is a noncompact manifold and $w : Y \to \mathbf{C}$ is a holomorphic function. The derived category of coherent sheaves of X is then expected to be equivalent to a derived category of Lagrangian vanishing cycles associated to the singularities of w. In the special case where the critical points of w are isolated and non-degenerate, a rigorous definition of the category of Lagrangian vanishing cycles has been given by Seidel, which makes the explicit verification of homological mirror symmetry possible on various examples.

The main example we will consider is the weighted projective plane $X = \mathbf{CP}^2(a, b, c) = (\mathbf{C}^3 - \{0\})/\mathbf{C}^*$, where a, b, c are positive integers and $\mathbf{C}^*$ acts by $t \cdot (x, y, z) = (t^a x, t^b y, t^c z)$. X is a Fano orbifold, and its mirror is the affine hypersurface $Y = \{x^a y^b z^c = 1\} \subset (\mathbf{C}^*)^3$ equipped with the superpotential $w = x + y + z$. By studying the vanishing cycles at the $a+b+c$ critical points of w, and putting them in relation with certain coherent sheaves on X, it is possible to prove homological mirror symmetry for weighted projective planes (joint work with L. Katzarkov and D. Orlov). In addition, it can be shown that non-exact symplectic deformations of Y correspond to certain noncommutative deformations of X. The same methods apply to other examples, such as Hirzebruch surfaces or certain Del Pezzo surfaces

References

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