# FAN 2010:

# Fluid dynamics, Analysis, and Numerics

A conference in honor of J. Thomas Beale



June 28-30, 2010

# Dukeuniversity

# 1 Introduction

Mathematical fluid dynamics, spanning many areas from rigorous analysis of nonlinear partial differential equations to numerical analysis to modeling and applied analysis of related physical systems, has provided a very fertile setting for long standing research in key areas:

- <u>Analysis of PDEs for fluid dynamics</u>: rigorous results for regularity, existence, uniqueness for Navier-Stokes and Euler, problems with free-surfaces and surface tension, convergence of vortex methods and splitting methods to solutions of the full problems
- <u>Fluid motion driven by interfaces</u>: computational methods and analysis of problems with moving interfaces
- <u>Computational methods for fluid dynamics</u>: advances in computational methods and applications

Thomas Beale has made important contributions to many problems in these areas. In recognition of his works, the FAN 2010 meeting brings together leading researchers to discuss current issues in fluid dynamics, analysis and numerics. We look forward to the meeting as a vibrant environment for thought-provoking exchanges spanning all aspects of fluid dynamics, analysis, and numerical methods. Warm welcome to all of the participants!

– The Organizing Committee

Thomas Witelski [Chair]	(Duke University)
David Ambrose	(Drexel University)
Andrea Bertozzi	(UCLA)
Anita Layton	(Duke University)
Zhilin Li	(North Carolina State University)
Michael Minion	(University of North Carolina at Chapel Hill)

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We gratefully acknowledge support from the conference sponsors:

- The National Science Foundation for participant travel support (NSF DMS 0963705)
- Department of Mathematics, Duke University
- Department of Mathematics, Drexel University
- Department of Mathematics, North Carolina State University
- Department of Mathematics, University of North Carolina at Chapel Hill

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- Multi-Scale Stochastic Finite Element Method (MSFEM) for stochastic partial differential equations

Mulin Cheng (Caltech)

• A numerical investigation of plasma expansion due to laser ablation using an efficient finitevolume method

Sean Cohen (North Carolina State University)

- Selected integration techniques for 2D Stokes flow Breschine Cummins and Mike Nicholas (*Tulane University*)
- Shallow-water surface waves and bed ripples due to erosion Matthew Emmett (*University of Alberta*)
- An all-speed asymptotic-preserving method for isentropic Euler and Navier-Stokes equations Jeffrey Haack (University of Wisconsin-Madison)
- Modeling and simulation of the fluid flow around the bell of the upside-down jellyfish Christina Hamlet (University of North Carolina-Chapel Hill)
- Solving matrix coefficient elliptic equations with sharp-edged interfaces Songming Hou (*Louisiana Tech University*)
- Lagrangian panel method for vortex sheet motion in 3D flow Robert Krasny (*University of Michigan*)
- Analysis of dynamics of Doi-Onsager phase transition Jian-Guo Liu (*Duke University*) (with Pierre Degond and Amic Frouvelle)
- Particle-laden viscous thin-film flows on an incline Nebojsa Murisic (UCLA)
- Fluid dynamics of the dinoflagellate transverse flagellum Hoa Nguyen (*Tulane University*)
- Gradient-augmented level set methods and interface tracking with subgrid resolution Benjamin Seibold (*Temple University*)
- Generalized Birkhoff-Rott equation for 2D active scalar problems Paul Hui Sun (*UCLA*) (with David Uminsky)
- A priori estimates for periodic viscous surface waves without surface tension Ian Tice (*Brown University*)
- Long-time behavior of weak solutions in Hele-Shaw flow problem Suleyman Ulusoy (*University of Maryland*)
- Exporting shock-capturing schemes from gas dynamics to elasticity Knut Waagan (University of Maryland)
- A mechanical and computational model of mucus penetration in mucociliary transport Xingzhou Yang (*Mississippi State University*)
- A fourth order Cartesian grid method for the Helmholtz equation with geometrically complicated material interfaces Shan Zhao (*University of Alabama*)

# 4 Monday abstracts

# Climate Science, Waves, and PDE's for the Tropics: Observations, Theory, and Numerics

# Andrew J. Majda

#### Courant Institute of Mathematical Sciences, New York University

Geophysical flows are a rich source of novel problems for applied mathematics and the contemporary theory of partial differential equations. The reason for this is that many physically important geophysical flows involve complex nonlinear interaction over multi-scales in both time and space so developing simplified reduced models which are simpler yet capture key physical phenomena is of central importance. In mid-latitudes, the fact that the rotational Coriolis terms are bounded away from zero leads to a strict temporal frequency scale separation between slow potential vorticity dynamics and fast gravity waves; this physical fact leads to new theorems justifying the quasi-geostrophic mid-latitude dynamics even with general unbalanced initial data for both rapidly rotating shallow water equations and completely stratified flows.

At the equator, the tangential projection of the Coriolis force from rotation vanishes identically so that there is no longer a time scale separation between potential vortical flows and gravity waves. This has profound consequences physically that allow the tropics to behave as a waveguide with extremely warm surface temperatures. The resulting behavior profoundly influences longer term mid-latitude weather prediction and climate change through hurricanes, monsoons, El Nino, and global teleconnections with the mid-latitude atmosphere. How this happens through detailed physical mechanisms is one of the most important contemporary problems in the atmosphere-ocean science community with a central role played by nonlinear interactive heating involving the interaction of clouds, moisture, and convection. The variable coefficient degeneracy of the Coriolis term at the equator alluded to earlier leads to both important new physical effects as well as fascinating new mathematical phenomena and PDE's. In this equatorial context, new multi-scale reduced dynamical PDE models are even relatively recent in origin.

After a brief discussion of the observational record as background, this lecturer surveys the remarkable new hyperbolic systems that have emerged recently in applications including their physical properties, applied mathematical and rigorous mathematical theory. These last topics include novel relaxation limits for climate models with active moisture and new singular limits for hyperbolic PDE's with variable coefficients. All of the references in this lecture can be found at http://www.math.nyu.edu/faculty/majda/.

# Steady Rotational Water Waves

# Walter Strauss

#### Department of Mathematics, Brown University

Consider a 2D gravity wave with arbitrary vorticity, possibly discontinuous. Suppose it is periodic and travels at a constant speed over a flat bed. By local and global bifurcation theory, there exist many such waves of large amplitude. Simulations illustrate certain relationships between the amplitude, energy and mass flux of the waves. Stagnation points may occur at points below the crest.

# Pattern formations of Benard-Marangoni heat convection problems

# Takaaki Nishida

Department of Mathematical Sciences, Waseda University

Oberbeck-Boussinesq system of equations is considered for the heat convection in the horizontal domain. The upper boundary is a free surface whose surface tension depends on the temperature. Depending on the parameters, both stationary and Hopf bifurcations are proved to occur from the equilibrium (heat conduction) state for this Benard-Marangoni problem.

# Novel phenomena in active and driven complex fluids

# Michael Shelley

Courant Institute of Mathematical Sciences, New York University

Fluids with suspended microstructure - complex fluids - are common actors in micro- and biofluidics applications and can have fascinating dynamical behaviors. I will overview some of the interesting phenomena and theoretical work in this area, but will concentrate largely on the dynamics of "active" suspensions of microswimmers such as bacteria. Such motile suspensions are important to biology, and are candidate systems for tasks such as microfluidic mixing and pumping. Experiments have shown transitions to persistent dynamics and fluid mixing on scales much larger than those of single swimmers. To understand these systems, we have developed both particle and continuum "complex fluid" models for studying the collective dynamics of hydrodynamically interacting microswimmers. The PDE model has a very interesting analytical and dynamical structure, and predicts critical conditions for the emergence of hydrodynamic instabilities. These predictions are verified in our detailed particle simulations, and are consistent with current experimental observation.

#### Multidimensional Aggregation Equations and Finite Time Blowup

# Andrea L. Bertozzi

#### Department of Mathematics, University of California Los Angeles

I consider kinematic aggregation equations in which a radially symmetric interaction kernel has a mild singularity at the origin. I will review recent results on this problem concerning well-posedness of nonnegative solutions and finite time blowup in multiple space dimensions, depending on the behavior of the kernel at the origin. Both analytical and numerical results will be presented including well-posedness in  $L^p$  spaces and the self-similar dynamics of blowup (including both dimensional and anomalous scaling).

# On the singularity formation of a 3D model for Incompressible Euler and Navier-Stokes equations

# Thomas Y. Hou

## Applied and Computational Mathematics, Caltech

We study the singularity formation of a recently proposed 3D model for the incompressible Euler and Navier-Stokes equations. This 3D model is derived from the axisymmetric Navier-Stokes equations with swirl using a set of new variables. The model preserves almost all the properties of the full 3D Euler or Navier-Stokes equations except for the convection term which is neglected. If we add the convection term back to our model, we would recover the full Navier-Stokes equations. We will present numerical evidence which supports that the 3D model may develop a potential finite time singularity. We will also analyze the mechanism that leads to these singular events in the new 3D model and how the convection term in the full Euler and Navier-Stokes equations destroys such a mechanism, thus preventing the singularity from forming in a finite time. Finally, we prove rigorously that the 3D model develops finite time singularities for a large class of initial data with finite energy and appropriate boundary conditions. This work may shed interesting light into the stabilizing effect of convection for 3D incompressible Euler and Navier-Stokes equations.

#### Moving interface problems for elliptic systems

# John A. Strain

#### Department of Mathematics, University of California Berkeley

Fluid dynamics and materials science often involve moving interfaces, with velocities determined by elliptic systems and interface geometry. Our implicit semi-Lagrangian contouring (iSLC) algorithm moves interfaces with arbitrary velocities, and separates model-dependent physics from interface motion. iSLC converts stiff moving interfaces to a grid-free contouring problem, and computes highly accurate solutions with merging, anisotropy, faceting, curvature and nonlocal interactions.

The interface velocity is computed by locally-corrected spectral (LCS) methods, which convert arbitrary elliptic problems to first-order overdetermined systems, mollify a periodic fundamental solution for convergence, and correct it via Ewald summation. The fundamental solution is employed in a boundary integral equation derived by local linear algebra and distribution theory. With a new geometric nonuniform fast Fourier transform, LCS methods provide efficient accurate solutions to elliptic systems with complex moving interfaces.

# **Existence Problems in Interfacial Fluid Dynamics**

# David M. Ambrose

Department of Mathematics, Drexel University

For many problems in interfacial fluid dynamics, such as the water wave or the vortex sheet with surface tension, existence of solutions for initial value problems and existence of traveling solutions has been established in recent years. In this talk, we will look at some other existence problems. First, we will explore the question of existence of weak solutions for interfacial Navier-Stokes flows, in the presence of surface tension. One significant challenge in this problem is finding a weak formulation of the surface tension force, which is supported only at the interface, and is given in terms of the curvature of the interface. As time allows, we will also discuss existence of time-periodic interfacial flows, as well as solutions to elliptic problems in interfacial fluid dynamics such as the unregularized vortex sheet and Boussinesq equations. This includes joint work with Milton Lopes Filho, Timur Milgrom, Helena Nussenzveig Lopes, Walter Strauss, and Jon Wilkening.

# 5 Tuesday abstracts

# Vanishing viscosity limits for a class of circular pipe flows, and related singular perturbation problems

# Michael E. Taylor

Department of Mathematics, University of North Carolina Chapel Hill

We consider 3D Navier-Stokes flows with no-slip boundary conditions in an infinitely long pipe with circular cross section. The velocity fields we consider are independent of the variable parametrizing the axis of the pipe, and the component of the velocity normal to the axis is arranged to be circularly symmetric, though we impose no such symmetry on the component of velocity parallel to the axis. For such flows we analyze the limit as the viscosity tends to zero, including boundary layer estimates. Carrying out this analysis leads naturally to consideration of related singular perturbation problems on bounded domains. This is joint work with Anna Mazzucato. (PDF files of three papers on this topic can be found on my web page.)

# Wellposedness of the two and three dimensional full water wave problem

# Sijue Wu

Department of Mathematics, University of Michigan

We consider the question of global in time existence and uniqueness of solutions of the infinite depth full water wave problem. We show that the nature of the nonlinearity of the water wave equation is essentially of cubic and higher orders. For suitably small initial data, we show that the 2-D full water wave equation is uniquely solvable almost globally in time, and the 3-D full water wave equation is uniquely solvable globally in time.

# On Accurate Methods for Field Interpolation in Particle Mesh Calculations

### Anita Mayo

Department of Mathematics, Baruch College

In this talk we will present accurate two and three dimensional methods for interpolating singular or smoothed force fields. The methods are meant to be used in particle mesh or particle-particle particle-mesh calculations so that the resulting schemes conserve momentum. The interpolation weights use a discretization of the differential equation the interaction force satisfies, and the methods are most accurate when the force satisfies a homogeneous elliptic differential equation or system of equations away from the sources. We will show why the precise accuracy of the interpolation formulas depends on the accuracy of certain corresponding quadrature formulas, and give results of numerical experiments.

# Well Balanced Methods for Conservation Laws with Source Terms

# Randall J. LeVeque

Department of Applied Mathematics, University of Washington

Hyperbolic conservation laws with a source term (balance laws) often require well-balanced methods that numerically preserve important steady state solutions in which the divergence of the flux exactly balances the source term. The goal is often to accurately compute solutions that are quasi-steady, modeling small amplitude waves propagating on top of such an equilibrium, for example when modeling tsunami propagation across the ocean. Examples also arise in a quasi-steady atmosphere in which the gravitational force is balanced by the pressure gradient. I will discuss a general approach to this problem in the context of high-resolution finite volume methods (Godunov-type methods) that are designed to also accurately capture discontinuous shock wave solutions.

# Immersed boundary methods for interfacial flows

# Ming-Chih Lai

## Department of Applied Mathematics, National Chiao Tung University

In this talk, numerical methods based on immersed boundary formulations are proposed for the simulations of two-dimensional capillary flows with interfaces. In particular, three different interesting physical problems are introduced; namely, (1) moving contact lines with surfactant; (2) von Neumann law and dry foam; and (3) the dynamics of inextensible vesicles. We shall discuss those numerical issues and report the simulation results.

# Solving the immersed interface problem using the decomposition with boundary integral approach

# Anita T. Layton

# Department of Mathematics, Duke University

We present a second-order accurate method for computing the coupled motion of a viscous fluid and an elastic material interface with zero thickness. The fluid flow is described by the Navier-Stokes equations, with a singular force due to the stretching of the moving interface. We decompose the velocity into three parts: a "Stokes" part, a "regular" part, and a "boundary correction" part. The "Stokes" part is determined by the Stokes equations and the singular interfacial forcer; thus, the Stokes solution is unsmooth. We first compute its values near the immersed interface using boundary integrals. Then those approximations are incorporated into the Poisson problems that are solved to yield second-order Stokes solutions. The regular part of the velocity is given by the Navier-Stokes equations with a body force resulting from the Stokes part, and with periodic boundary conditions. The regular velocity is obtained using a time-stepping method that combines the semi-Lagrangian method with the backward difference formula. Because the body force is continuous, jump conditions are not necessary. The boundary correction solution is described by the unforced Navier-Stokes equations, with Dirichlet boundary conditions given by the difference between the Dirichlet boundary conditions of the overall Navier-Stokes solution, and the boundary values of the Stokes and regular velocities. Because the boundary correction solution is sufficiently smooth, jump conditions are also not necessary. Numerical results exhibit approximately second-order accuracy in time and space.

# Implicit particle filters with applications to ocean data

#### Alexandre J. Chorin

# Department of Mathematics, University of California Berkeley

Particle filters are needed for data assimilation in many applications, for example in economics and in meteorology, but they often fail because it is difficult to find high probability samples. Implicit sampling solves this problem by first picking probabilities and then finding samples to assume them; in so doing it guides the particles one by one to the high probability domain without requiring a global trial density. I will explain the problem and its solution from scratch, and provide examples. (joint work with X. Tu and M. Morzfeld)

# 6 Wednesday abstracts

# Lagrangian blob methods applied to biological fluid flow problems

# **Ricardo Cortez**

# Department of Mathematics, Tulane University

Biological flows, such as those surrounding swimming organisms or beating cilia, can be properly modeled using Lagrangian methods for fluid motion with external forcing. The organism surfaces can be viewed as flexible interfaces imparting force or torque on the fluid. Examples of filaments or structures swimming in a fluid based on Lagrangian methods that use regularization (blobs) will be presented. The methods are based on integral expressions for the exact fluid velocity field resulting from localized forces supported in small spheres, rather than point-forces. I will present the idea of the methods, some of the known results and improvements obtained from using corrections proposed by Tom Beale.

# Stabilizing fluid-fluid interfaces using colloidal particles

# John S. Lowengrub

## Department of Mathematics, University of California - Irvine

Bicontinuous interfacially jammed emulsion gels ('bijels') were proposed in 2005 as a hypothetical new class of soft materials in which interpenetrating, continuous domains of two immiscible fluids are maintained in a rigid state, by a jammed layer of colloidal particles at their interface. Such gels offer an important route to materials with unique combinations of properties not available in a single phase material. In 2007, the first bijels were created experimentally. In joint work with Sebastian Aland and Axel Voigt, we develop a continuum model for such systems which combines a Cahn-Hilliard-Navier-Stokes model for the macroscopic two-phase fluid system with a surface Phase-Field-Crystal model for the microscopic colloidal particles along the interface. We demonstrate the feasibility of this approach and present numerical simulations that confirm the ability of the colloids to stabilize interfaces for long times.

#### **Blob Methods for Free surface Flows**

# Gregory R. Baker

# Department of Mathematics, Ohio State University

Blob methods were designed to restore regularity to the motion of vortex sheets, but they may also be viewed as a technique to approximate boundary integrals that arise in free surface flows. The blob method smooths the singularity in the integrand of the principal-valued integrals, and standard numerical integration may then be applied to the result. There is a balance between errors introduced by smoothing and the numerical integration that can be refined to design very accurate methods. In particular, results are comparable to spectral accuracy in two-dimensional geometry. However, it is in three-dimensional geometry that blob methods may prove to be the most powerful. Results will be presented that demonstrate the accuracy and performance of blob methods, as well as their application to the motion of water waves in both two and three dimensions.

# 7 Poster abstracts

• Using regularized Stokeslets to model inextensible fibers in Stokes flow Elizabeth Bouzarth (*Duke University*)

The behavior of inextensible fibers immersed in a fluid is of interest in a variety of applications ranging from polymer suspensions to actin filament transport. In these cases, the dynamics of an immersed fiber can play a large role in the observed macroscale fluid dynamics. The method of regularized Stokeslets provides a way to calculate fluid velocities in the Stokes fluid flow regime due to a collection of regularized point-forces without computing fluid velocities on an underlying grid. In this work, the method of Regularized Stokeslets is used to model the dynamics of an inextensible flexible fiber immersed in a two-dimensional cellular background flow. Studying this fluid flow scenario with regularized Stokeslets provides insight into the documented stretch-coil transition and macroscale random walk behavior supported by mathematical models and experimental results.

• Multi-Scale Stochastic Finite Element Method (MSFEM) for stochastic partial differential equations Mulin Cheng (*Caltech*)

Uncertainty arises in many fluid problems, such as wave, heat and pollution propagation through random media, randomly forced Burgers or Naiver-Stokes equations and etc, many of which can be described concisely by Stochastic Partial Differential Equations (SPDEs). However, the numerical computation of their solutions poses a great challenge to the computational fluid dynamic (CFD) community. Wiener Chaos Expansion method (WCE) and their variants emerging in recent years show some promising features but still suffer greatly from the slow convergence of polynomial expansion and the curse of dimensionality. On the other side, the convergence rate of Monte Carlo method (MC) is independent of the number of stochastic dimensions, but it is merely  $O(1/\sqrt{N})$ . The key observation leading to this paper is that the basis used in WCE is determined a prior and problem-independent, which, we believe, is the ultimate reason of non-sparsity. Partially inspired by Multi-scale Finite Element methods and Proper Orthogonal Decomposition methods, we proposed here an innovative algorithm, which consists of two parts, offline and online, and fuses both MC and WCE. In the offline part, a set of nonlinear stochastic basis  $\{A_i(t,\omega)\}$  are constructed based on the Karhunen-Loeve(KL) expansion, which can be viewed as partially inversion of random differential operator. In the online part, polynomial basis used in WCE are replaced by the constructed stochastic basis and the solution is represented as an expansion under the nonlinear stochastic basis. By solving a set of coupled PDEs of coefficients, we obtain the numerical solutions to SPDEs. The proposed framework is applied to 1D and 2D elliptic problems with random coefficients. Numerical examples show that the stochastic basis obtained in the offline computation can be repeated used in online computation for different scenario and much smaller number of basis are necessary in the online computation to achieve certain error tolerance compared to WCE.

• A numerical investigation of plasma expansion due to laser ablation using an efficient finitevolume method

Sean Cohen (North Carolina State University)

In this paper we investigate the laser ablation model presented in (Trofimov, et al., Tech. Phys. **53** (2008), no. 2, 154-159 and Tech. Phys. **54** (2009), no. 7, 974-980). In laser ablation, a laser pulse strikes a solid material thereby causing fast evaporation of the target material which then expands into the surrounding atmosphere as a plasma. Efficient computation is important since the physical experiments are difficult to implement. The model used is governed by convection-diffusion equations. The stiffness of the diffusive terms and the evolution of the gas dynamics present difficulties to numerical schemes. The finite volume method used is the central-upwind scheme developed in (Kurganov et al. SIAM J. Sci. Comput. **23** (2001), no. 3, 707-740). The scheme is able to produce high-resolution results at extensive computational savings allowing for the investigation of model parameters and their effects on results as well as the development of multi-dimensional schemes.

- Selected integration techniques for 2D Stokes flow Breschine Cummins and Mike Nicholas (*Tulane University*)
- Shallow-water surface waves and bed ripples due to erosion Matthew Emmett (*University of Alberta*)

Deformations in the bed of a river or flume that are caused by erosion may eventually lead to the formation of ripples in the bed and surface roll-waves on the water. The complex interaction between the flow and bed that produces these phenomena is quite intriguing and we endeavor to understand the mechanisms behind it. To this end, we present a simple bulk model of sediment transport and erosion, and explore, using various analytic and asymptotic techniques, how the numerically observed roll-waves and bed ripples are formed.

• An all-speed asymptotic-preserving method for isentropic Euler and Navier-Stokes equations Jeffrey Haack (University of Wisconsin-Madison)

The computation of compressible flows becomes more challenging when the Mach number has different orders of magnitude. When the Mach number is of order one, modern shock capturing methods are able to capture shocks and other complex structures with high numerical resolutions. However, if the Mach number is small, the acoustic waves lead to stiffness in time and excessively large numerical viscosity, thus demanding much smaller time step and mesh size than normally needed for incompressible flow simulation. In this paper, we develop an all-speed asymptotic preserving (AP) numerical scheme for the compressible isentropic Euler and Navier-Stokes equations that is uniformly stable and accurate for all Mach numbers. Our idea is to split the system into two parts, one involves a slow, nonlinear and conservative hyperbolic system adequate to use modern shock capturing methods, and the other a linear system which contains the stiff acoustic dynamics, to be solved implicitly. This implicit part is turned into a standard pressure Poisson projection step, thus possesses a sufficient structure for efficient fast Fourier transform solution techniques. In the zero Mach number limit, the scheme automatically becomes an projection method like incompressible solver. We present numerical results in one and two dimensions in both compressible and incompressible regimes. • Modeling and simulation of the fluid flow around the bell of the upside-down jellyfish Christina Hamlet (University of North Carolina-Chapel Hill)

Pulsatile jet propulsion is one of the earliest forms of muscle-driven locomotion. Jellyfish use contractions of their bell to form vortex rings that facilitate locomotion. The upside-down jellyfish (genus Cassiopea) which utilize zooanthellae for photosynthetic feeding, also exhibit pulsatile motion engaged in particle exchange generally uncoupled from swimming. Here we present qualitative data demonstrating how these undulations are used for particle transfer. Immersed boundary methods are used to simulate bell pulses of a 2D jellyfish and explore the effect of parameter variation on the structure of flow around the model organism.

• Solving matrix coefficient elliptic equations with sharp-edged interfaces Songming Hou (*Louisiana Tech University*)

Solving elliptic equations with sharp-edged interfaces is a challenging problem for most existing methods, especially when the solution is highly oscillatory. Nonetheless, it has wide applications in engineering and science. I will present a non-traditional finite element method for solving matrix coefficient elliptic equations with shape-edged interfaces with 2nd order accuracy in L-infinity norm.

• Lagrangian panel method for vortex sheet motion in 3D flow Robert Krasny (*University of Michigan*)

A Lagrangian panel method is presented for computing vortex sheet motion in 3D flow. The sheet is represented by a set of quadrilateral panels with a quadtree structure. The panels have active particles that carry circulation and passive particles used for adaptive refinement. The Biot-Savart kernel is regularized and the particle velocity is computed using a treecode. Results are presented for the azimuthal instability of a vortex ring and the oblique collision of two vortex rings. This is joint work with Hualong Feng and Leon Kaganovskiy and some of the results have appeared in Fluid Dynamics Research, vol. 41 (2009). Preliminary results with Lei Wang for the barotropic vorticity equation on a rotating sphere will also be presented.

- Analysis of dynamics of Doi-Onsager phase transition Jian-Guo Liu (*Duke University*) (with Pierre Degond and Amic Frouvelle)
- Particle-laden viscous thin-film flows on an incline Nebojsa Murisic (*UCLA*)

We consider gravity driven flows of particle-laden thin films on an incline. Three distinct regimes are observed depending on the inclination angle and the bulk volume fraction of particles: the particles either settle out of the flow, aggregate at the moving front, or remain well-mixed. The experiments are carried out with a range of particle sizes and fluid viscosities. The results compare well with an equilibrium theory balancing shear-induced migration with settling in the normal direction. We find that the well-mixed regime is transient in nature, with both the particle size and the liquid viscosity affecting the time scale on which it occurs. • Fluid dynamics of the dinoflagellate transverse flagellum Hoa Nguyen (*Tulane University*)

The action of the transverse flagellum encircling the dinoflagellate body is investigated from a hydrodynamical point of view. The flagellum is modeled as a closed circular helix and its self-propulsion is achieved by propagating waves along its length. Waves that propagate counter-clockwise along the flagellum give rise to its clockwise motion as well as forward thrust. The grid-free method of regularized Stokeslets is used to understand the fluid dynamics of the flagellum at low Reynolds number. The resulting motion is also analyzed by slender-body theory.

• Gradient-augmented level set methods and interface tracking with subgrid resolution Benjamin Seibold (*Temple University*)

Level set methods represent an interface implicitly as a contour of a level set function, which is defined on an Eulerian grid. The grid values are evolved in time such that the correct interface movement is recovered. This is frequently done by high order ENO/WENO schemes. Key advantages of level set methods (compared to explicit approaches) are robustness, and a simple treatment of topology changes. Challenges are the "loss of mass" due the vanishing of small or thin structures when falling below the grid resolution, the construction of high order WENO stencils near boundaries, and the accurate approximation of curvature.

We show how these problems can be ameliorated when gradients of the level set function are evolved as independent quantities. First, through an appropriate Hermite interpolation, a certain level of subgrid resolution is achieved. Second, by the use of the method of characteristics, an accurate, optimally local gradient-augmented scheme can be formulated. Third, the interpolation yields an accurate approximation to the curvature. The presented method has the simplicity of fully Eulerian approaches, and its optimal locality admits a simple combination with adaptive mesh refinement techniques. The new approach is used to track the interface in two-phase fluid dynamics simulations. It turns out that compared to classical WENO schemes, the representation of small structures is improved.

(Joint work with Rodolfo Ruben Rosales and Jean-Christophe Nave)

• Generalized Birkhoff-Rott equation for 2D active scalar problems Paul Hui Sun (*UCLA*) (with David Uminsky)

In this poster we derive new evolution equations for the active scalar problem in 2D for the case when all scalars lie on a 1D curve, analogous to the Birkhoff-Rott equation for 2D vorticity. The new equations are Lagrangian and valid for nonlocal kernels K that may include both a gradient and an incompressible term. We develop a numerical method for implementing the model which achieves second order convergence in space and fourth order in time. We simulate several classic vortex sheet examples (in the case of a purely incompressible kernel) and the collapse of delta ring solutions (in the case of a purely gradient kernel) and find excellent agreement with our new model. We then analyze two examples that include both incompressible and gradient parts, the first is a model for superfluids and the second a model for collective biological motion and discuss the results. • A priori estimates for periodic viscous surface waves without surface tension Ian Tice (*Brown University*)

We consider the incompressible gravity-driven Navier-Stokes equations in a three-dimensional slab domain with a free upper boundary. The domain is taken to be horizontally periodic and of finite vertical depth, with lower boundary given by a smooth periodic function b and with upper boundary given as a moving free surface. We are concerned with a priori estimates for Sobolev norms of the solution and its time derivatives in an appropriately flattened coordinate system. We utilize a nonlinear energy method in order to show that if the initial data is sufficiently small (in terms of an energy defined via various Sobolev norms), then high derivatives remain bounded while lower order derivatives actually decay in time.

• Long-time behavior of weak solutions in Hele-Shaw flow problem Suleyman Ulusoy (*University of Maryland*)

We investigate the long-time behavior of weak solutions to the thin- film type equation  $u_t = -(uu_{xxx})_x$ . We employ a semidiscrete variational scheme to generate weak solutions as a gradient flow with respect to so called Wasserstein distance and we show that these weak solutions converge to the unique self-similar source type solution exponentially fast. This paper complements our results in Carlen, E. A., Ulusoy, S. : Asymptotic equipartition and long time behavior of solutions of a thin-film equation, J. Differential Equations, 241, pp. 279-292, 2007. This is a joint work with Eric A. Carlen.

• Exporting shock-capturing schemes from gas dynamics to elasticity Knut Waagan (*University of Maryland*)

Elastic materials are modeled by nonlinear hyperbolic conservation laws with complicated constitutive equations. Efficient numerical techniques have been developed for shocked gases. We consider the application of such shock-capturing schemes to nonlinear elasticity. These schemes add dissipative mechanisms that lack proper invariance under rigid motions, which is shown to lead to large errors. We present a method for imposing invariant dissipative mechanisms. The result is a rich class of invariant regularizations which may yield new insight on shock waves in solids, both numerically and theoretically.

• A mechanical and computational model of mucus penetration in mucociliary transport Xingzhou Yang (*Mississippi State University*)

Mucociliary transport is a complex dynamical process that serves as defense mechanism in the lung. In this process, mucus penetration, a dynamical interaction of cilia and mucus layers with their surrounding fluid, plays important roles. Dysfunctions in mucociliary clearance will cause several lung diseases. We propose a mechanical and computational model to simulate this complex system. This model couples the time-dependent fluid dynamics and the internal force generation algorithm by ATP-induced molecular motor proteins.

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• A fourth order Cartesian grid method for the Helmholtz equation with geometrically complicated material interfaces Shan Zhao (*University of Alabama*)

Across the dielectric interfaces, the electromagnetic wave solutions are usually non-smooth or even discontinuous, so that our effort in designing efficient algorithms is easily foiled, unless the complex interfaces are properly treated. We have recently introduced a novel higher order finite difference method – the matched interface and boundary (MIB) method, for solving the Helmholtz equation with arbitrarily curved dielectric interfaces based on a simple Cartesian grid. Like other Cartesian grid methods, the MIB method in some sense fits the numerical differentiation operators to the complicated geometries. Nevertheless, the MIB method distinguishes itself from the existing interface methods by avoiding the use of the Taylor series expansion and by introducing the concept of the iterative use of low order jump conditions. The difficulty associated with other interface approaches in extending to ultra high order is thus bypassed in the MIB method. In solving waveguides with straight interfaces, the MIB interface treatment can be carried out systematically so that the proposed approach is of arbitrarily high order, in principle. Orders up to 12 are confirmed numerically for both transverse magnetic (TM) and transverse electric (TE) modes. In solving waveguides with curved interfaces, the enforcement of jump conditions couples two transverse magnetic field components, so that the resulting MIB method becomes a full vectorial approach. The full vectorial MIB method has been shown to be able to deliver a fourth order of accuracy in treating arbitrarily curved interfaces.