

Duke University Math News

September 20, 2002

Welcome

Welcome to the Duke Mathematics Department! I want to extend a special welcome to all of our newcomers – faculty and students alike – as well as returning faculty, staff and students.

Dick Hain finished a three-year term of service as Department Chair on August 31, and I took over as the new Chair on September 1. I am fortunate to have Mike Reed continuing as Associate Chair and Les Saper continuing as Director of Graduate Studies. The administrative team is rounded out by Arlie Petters who has just assumed the job of Director of Undergraduate Studies. All four of us look forward to working with you.

We are in the third year of our department's VIGRE grant, with VIGRE activities now being managed by Harold Layton, who took over this task from Andrea Bertozzi. These activities affect the department at all levels, and have enabled us to bring a very exciting group of programs to you. We have a large number of postdocs in the department this year, many supported by VIGRE; we have undertaken an expansion of our graduate course offerings under the auspices of this program, and support for many of our graduate students is provided by it. Our PRUV program (part of VIGRE) provides research opportunities for undergraduate students, and the CHISEL program (also part of VIGRE) gives us an opportunity for outreach to the broader community.

Our physical surroundings changed again over this past summer, with the closure of Science Drive and the construction of the traffic circle adjacent to our building. This has made it easier to cross the street than it used to be, so I hope you'll take advantage of the opportunity to come by the department and get involved in some of the many activities here!

—Dave Morrison, Chair

Events

Online Workshop on Interactive Web-Based Mathematics

How can you use an online map of Virginia to estimate the area of the state? One answer is contained in web-based materials created by a team from Hollins University during an online workshop run from the Duke Mathematics Department, July 16–19. Associate professors Lang Moore and David Smith from the Duke and Frank Wattenberg from West Point conducted the workshop, *Authoring Online Interactive Materials in Mathematics*, for sixteen teams of 2 to 4 participants at institutions in the United States and Europe. Presentations by the instructors covered basics of HTML, creation of web pages using *Word* and *Dreamweaver*, working with applets and graphics, and the basics of *Maple*. The participants learned to create interactive, web-based materials for undergraduate mathematics.

Systems Programmer Andrew Schretter set up and maintained communication among the instructors and the participants by employing a variety of free software and the judicious use of long-distance telephoning. The online presentations were recorded and are available for view at the workshop web site: www.math.duke.edu/education/prep02. Many of the projects created by the teams during the workshop are also available at this site.

This workshop was the first of the Mathematical Association of America (MAA) PREP Workshop Program. A particular focus was the use of easily modifiable applets created as part of the Lite Applets Project, a component of MAA's Mathematical Sciences Digital Library (MathDL). The area-of-Virginia module uses a Lite applet that enables the user to transfer screen coordinates of points on a graphic to a spreadsheet. Wattenberg directs the Lite Applets Project, Smith is Editor of the *Journal of*

Online Mathematics and its Applications (www.joma.org), the journal of MathDL, and Moore is Executive Editor of MathDL.

Geometry Festival

Duke University will host the 18th annual Geometry Festival from 13–16 March 2003. This conference will be held in honor of Phillip Griffiths, the director of the Institute for Advanced Study in Princeton. Griffiths served as provost and James B. Duke Professor of Mathematics at Duke from 1983–1991. Duke previously hosted the Geometry Festival in 1991 and 1997.

Undergraduate News

DUMU

The Duke University Math Union, (DUMU), organizes activities including social events, such as picnics, movies, frisbee games with the Society for Physics Students, and our high-school math contest. Additionally, we invite speakers for the entertaining and informative Undergraduate Lecture Series. If you are interested in hearing about DUMU events and are not already on our mailing list, contact David Arthur at dga2@duke.edu. Read on to find out about our current plans.

The High School Math Meet

DUMU expects to host a contest this fall for high schools in North Carolina and nearby states.

We need problems and solutions for it. The more people who contribute, the more varied and interesting the contest will be. So, start thinking, and keep your eyes open for intriguing ideas. Math may be a tool and a subject, but it can also be a sport, a game, a kind of art, and thought-provoking fun.

We are looking for original problems of varying difficulty that can be solved in 2–10 minutes using pre-calculus mathematics only. The problems most likely to be used are those that reward insight more than experience. We will also need a few very easy problems.

Send submissions to David Arthur at dga2@duke.edu. See <http://www.math.duke.edu/dumu> for more information about the contest and last year's problems.

Competitions

The following contests are open to all undergraduates. They take place on Saturdays and are held in a math classroom in the Physics Building. If you are interested in participating in one of these competitions, or if you would just like more information, contact David Morrison at drm@math.duke.edu.

- The Virginia Tech Regional Math Contest will be held Saturday at 9:00 A.M. October 26. Last year, 269 students from 35 schools throughout the southeast, including 16 from Duke, competed in this two and a half hour contest of mathematical ingenuity and problem solving ability. In that contest, David Arthur '04, Kevin Lacker '02 and Oaz Nir '05 finished second, third and fourth. See <http://www.math.vt.edu/events/> for more information and past tests.
- The William Lowell Putnam Mathematical Competition will be held on Saturday December 7. Last year, nearly 3000 undergraduates at over 400 colleges in the US and Canada competed for team and individual awards. The Duke team of David Arthur '04, Nathan Curtis '02, and Kevin Lacker '02 placed third. Melanie Wood '03 won the Elizabeth Lowell Putnam award for the highest score among participating females. For more information, see <http://math.scu.edu/putnam/>.

PRUV Fellows

Undergraduates interested in mathematical research are encouraged to apply for the NSF sponsored summer PRUV program. Up to eight sophomores and juniors will be matched with Duke mentors for six weeks during the summer of 2003. Each student's research should lead to an undergraduate thesis worthy of Graduation with Distinction. The stipend will be at least \$3000.

The current PRUV Fellows, Tom Bringley, Ben Cook, Eric Fountain, Marie Guerraty, Meredith Houlton, David Marks, Steve Nicklas, and Dane Voris, will continue their research in independent study courses this year. For more information, contact David Kraines at dkrain@math.duke.edu or see www.math.duke.edu/vigre/pruv/.

Graduate Program News

New Graduate Students

- Michael Gratton, Harvey Mudd College
- Manuel Hagoulon, University of Paris
- John Hyde, Duke University
- Thomas Laurent, Paris, Duke University
- William LeFew, Northwestern University
- Maijaleena Low, University of Oregon
- James Michael, Dalhousie University
- Joseph Spivey, Emory University
- Jeffrey Streets, University of Chicago
- Philip Vetter, Princeton University
- Song Xiao, Beijing University

Graduate Courses

New and Topics courses for Fall 2002

1. **Math 215** Mathematical Finance: Option Pricing and Portfolio Theory (Xiaoying Dong). Introduction to basic concepts in mathematical finance from both theoretical and practical points of view, applying tools from optimization theory, stochastic calculus and stochastic differential equations. Topics include American and European options, geometric Brownian motion, Ito's lemma, Black-Scholes model of options; time value of money, rate of return of an investment cash-flow sequence, utility functions, mean-variance analysis of portfolios, efficient frontiers and the Capital Assets Pricing Model.
2. **Math 236/Physics 292** General Relativity (Arlie Petters). Introduction to the basic concepts and techniques of General Relativity. The course covers the fundamentals of tensor calculus, Lorentzian geometry, and Einstein's equations, as well as applications to cosmology, black holes, and gravitational lensing.
3. **Math 268** Topics in Differential Geometry (Mark Stern). This course will be an introduction to the Seiberg-Witten equations and the moduli spaces of their solutions. This will involve studying aspects of gauge theory, spin geometry, and complex geometry.
4. **Math 277** Topics in Algebraic Geometry (Dave Morrison). Calabi-Yau manifolds and their use in mathematical physics. Specific topics will include: holonomy of Riemannian manifolds, statement of Yau's theorem, complex projective varieties, Kähler geometry, constructions of varieties with trivial canonical bundle, period maps, linear and non-linear sigma models, Calabi-Yau compactifications in string theory.
5. **Math 283** Topics in Functional Analysis and PDE (Mike Reed). This course will cover a variety of analysis topics not usually covered in depth in our curriculum, such as Scattering Theory, Distribution Theory, Microlocal Analysis, Theory of Semilinear hyperbolic PDEs, Non-linear harmonic analysis.
6. **Math 295** Computational Geometry from a Mathematical Perspective (John Harer). The instructor will cover geometric techniques for data analysis in the context of certain kinds of applications. These include terrain analysis via GIS (the mathematics behind it), some molecular modeling issues, parts positioning systems and motion planning. The theme will be to describe these problems geometrically and then to discuss

topologically robust methodologies for solving the problems computationally.

Students can find out more about these and other graduate courses being offered this term at http://www.math.duke.edu/graduate/grad_courses.html.

—Les Saper, Director of Graduate Studies

Faculty News

New Faculty

The following professors and research associates have joined the department this fall.

- **Alvard Arazyan** (PhD Univ. of Louisiana at Lafayette), Instructor. *Statistics*.
- **Paula Budu** (PhD Univ. Durham, UK), Research Associate. *Mathematical Biology*.
- **Hyung Ju Hwang** (PhD Brown University), Assistant Research Professor. *Applied Mathematics, Fluid Flow*. (on leave 2002-03)
- **Daniel Marthaler** (PhD Arizona State), Research Associate. *Dynamical Systems and Chaos*.
- **Jonathan Mattingly** (PhD Princeton), Assistant Professor. *Stochastic Differential Equations*. (on leave 2002-03)
- **Lawrence Myers** (PhD Tulane), Instructor. *Statistics*.
- **Svetlana Roudenko** (PhD Michigan State Univ.), Assistant Research Professor. *Harmonic Analysis*.
- **Linda Smolka** (PhD Penn. State), Research Associate. *Non-newtonian Fluid Flow*.
- **Chad Topaz** (PhD Northwestern), Research Associate. *Nonlinear Dynamics and Pattern Formation*.
- **Gregor Weingart** (PhD Bonn Univ.), Research Associate. *Differential Geometry*.

Spring 2003:

- **Vladimir Itskov** (PhD Univ. Minnesota), Visiting Assistant Professor. *Differential Equations and Differential Geometry*.
- **Liping Liu** (PhD Univ. Alberta), Instructor. *Applied Mathematics* and Research Associate. *Mechanical Engineering*.
- **Sandra Wieland** (PhD Bonn Univ.), Visiting Scholar. *Applied Mathematics, Thin Films*.

Bryant elected to AAAS

Robert Bryant, J. M. Kreps professor of mathematics, was elected to the American Academy of Arts and Sciences last spring. Founded in 1780, The AAAS is an international learned society composed of the world's leading scientists, scholars, artists, business people, and public leaders. The 3,700 American Fellows include about 200 mathematicians. Bryant is the first math professor at Duke to become a member of this elite organization.

Bryant is the chair of the board of Trustees of the Mathematical Sciences Research Institute, serves on the Executive Committee of the Council of the American Mathematical Society, is the chair of the AMS Committee on Publications and is on the editorial boards of the *Duke Mathematical Journal* and of *Differential Geometry and Its Applications*. Last April, he was appointed by President Bush to serve on the Board of Directors of the Vietnam Education Foundation.

Bryant received his PhD in mathematics in 1979 at the University of North Carolina at Chapel Hill. Before coming to Duke in 1987, he was an NSF Postdoctoral Research Fellow, Alfred P. Sloan Fellow, Presidential Young Investigator and professor at Rice University.

Blackwell–Tapia Prize

Arlie Petters, William and Sue Gross associate professor of mathematics, is the first recipient of the Blackwell-Tapia Prize. This award, established by the Mathematical Sciences Research Institute and Cornell University in honor of David Blackwell and Richard

A. Tapia, is presented to a mathematical scientist who has contributed significantly to his or her field of expertise, and who has served as a role model for mathematical scientists and students from under-represented minority groups or contributed in other significant ways to the addressing of the problem of the under-representation of minorities in mathematics. See <http://www.msri.org/people/staff/meggin/pettersannounce.html> for more information.

Petters received his doctorate from MIT in 1991 and joined the Duke Math Department in 1998 after positions at MIT, and Princeton University. He is currently the Director of Undergraduate Studies.

Problem Corner

Solutions from Last Issue

- Problem.** If the sum of the coefficients of polynomials $p(x)$ and $q(x)$ are both 1, prove that the sum of the coefficients of $p(q(x))$ is also 1.

Solution. Since the sum of the coefficients of $p(x)$ and $q(x)$ are both 1, $p(1) = q(1) = 1$. Therefore, $p(q(1)) = 1$, and hence the sum of the coefficients of $p(q(x))$ is 1.

- Problem.** Find all real numbers, x and y , satisfying

$$\begin{aligned} x &= 11 - \frac{50y}{x^2 + y^2} \\ y &= 10 - \frac{50x}{x^2 + y^2} \end{aligned}$$

Solution. Multiply the second equation by i and add to obtain

$$\begin{aligned} x + yi &= 11 + 10i + \frac{50(-xi - y)}{x^2 + y^2} \\ &= 11 + 10i + \frac{50i(-x + yi)}{x^2 + y^2} \\ &= 11 + 10i - \frac{50i}{x + yi} \end{aligned}$$

Letting $z = x + yi$, and simplifying, we have

$$\begin{aligned} z^2 - (11 + 10i)z + 50i &= 0 \\ \Rightarrow (z - 8 - 6i)(z - 3 - 4i) &= 0 \end{aligned}$$

Since x and y are real, it follows that $x = 8$, $y = 6$ or $x = 3$, $y = 4$. It is easy to check these are both valid solutions.

- Problem. Putnam 2001.** In a triangle of area 1, each of three lines from the vertices to the opposite sides are bisected by another of these lines forming an interior triangle. Find the area of this smaller triangle.

Solution. Choose r, s, t so that $EC = rBC$, $FA = sCA$, $GB = tCB$, and let $[XYZ]$ denote the area of triangle XYZ . Then $[ABE] = [AFE]$ since the triangles have the same altitude and base.

Also $[ABE] = (BE/BC)[ABC] = 1 - r$, and $[ECF] = (EC/BC)(CF/CA)[ABC] = r(1 - s)$ (e.g., by the law of sines). Adding this all up yields

$$\begin{aligned} 1 &= [ABE] + [ABF] + [ECF] \\ &= 2(1 - r) + r(1 - s) = 2 - r - rs \end{aligned}$$

or $r(1+s) = 1$. Similarly $s(1+t) = t(1+r) = 1$.

Let $f : [0, \infty) \rightarrow [0, \infty)$ be the function given by $f(x) = 1/(1+x)$; then $f(f(f(r))) = r$.

However, $f(x)$ is strictly decreasing in x , so $f(f(x))$ is increasing and $f(f(f(x)))$ is decreasing. Thus there is at most one x such that $f(f(f(x))) = x$; in fact, since the equation $f(z) = z$ has a positive root $z = (-1 + \sqrt{5})/2$, we must have $r = s = t = z$.

We now compute $[ABF] = (AF/AC)[ABC] = z$, $[ABR] = (BR/BF)[ABF] = z/2$, analogously $[BCS] = [CAT] = z/2$, and

$$\begin{aligned} [RST] &= |[ABC] - [ABR] - [BCS] - [CAT]| \\ &= \left| 1 - \frac{3z}{2} \right| = \frac{7 - 3\sqrt{5}}{4}. \end{aligned}$$

New Problems

1. A coin is flipped repeatedly until either two consecutive heads or two consecutive tails occur. Given that the first time that the coin is flipped, it is heads, what is the probability that two consecutive heads will occur before two consecutive tails?

2. What polynomials, p , satisfy

$$p(x + y) + p(y + z) + p(z + x) = p(x + y + z) + p(x) + p(y) + p(z)$$

for all x, y, z ?

3. Prove that in any graph G containing at least one edge, there exist 2 vertices with the same degree, at distance at most 2 apart (Sasha Schwartz).

Submit solutions or suggestions for new problems to Problem Editor Dave Arthur dga2@duke.edu

Duke Math News

The *Duke Math News* is published several times a year and is distributed to those in the Duke mathematics community by campus mail. For previous editions and other news, see www.math.duke.edu/news/. We welcome items of interest for our next issue. Send them to splayer@math.duke.edu or dkrain@duke.edu

To read about other news, honors and events concerning mathematics at Duke, visit www.math.duke.edu/news/. The on-line calendar at www.math.duke.edu/mcal lists both regular and special seminars and colloquia for the upcoming weeks. The department maintains video archives of talks, lecture series and special conferences at Duke, many of which are available, on-line. See www.math.duke.edu/computing/broadcast.html for more information.

—David Kraines, *DMN Faculty Sponsor*

Problem Corner Editor

David Arthur dga2@duke.edu

Faculty Sponsor

David Kraines dkrain@math.duke.edu

Production Manager

Sara Player splayer@math.duke.edu

Department of Mathematics

Box 90320

Durham, NC 27708-0320

http://www.math.duke.edu/math_news/