Duke University Math News

October 5, 1999

Welcome

We hope that you had a productive summer and that you have found exciting and challenging courses to take this fall.

Several changes among the department officers have taken place over this summer. Former chair, John Harer, is now the Vice Provost for Academic Affairs. The new chair, Professor Richard Hain, joined the Duke math department in 1991 after 10 years at the University of Washington. Hain received his Bachelor of Science and Masters degree in his native Australia and his PhD at the University of Illinois. He has published extensively in the fields of topology and algebraic geometry and supervised several PhD and undergraduate theses. Professor William Pardon will continue as Associate Chair and Professor J. Thomas Beale as Director of Undergraduate Studies. Professor William Allard will become Director of Graduate Studies.

W. Garrett Mitchener, editor of the Duke Math News during his entire four years at Duke, is now working on his doctorate in applied mathematics at Princeton University. The new editor of this newsletter, senior Carl Kingsford, will be assisted by freshman Robert Morris who will write features about math at Duke. Rob welcomes suggestions for topics of interest.

Duke University Math Union (DUMU) president, junior Carl Miller, needs volunteers to help organize DUMU's popular ARML style math meet on Saturday, November 13. If you can help, send him a note at carl@math.duke.edu. If your old high school is within a 5 hour drive from Duke, encourage them to send a team. Last year's event attracted about 100 high school students from throughout the state and elsewhere for a fun afternoon contest. Check out http://www.math.duke.edu/dumu/ for details and for other upcoming DUMU events.

Share with us any news items of general interest. Did you have a summer job that made use of your mathematical training? Have you received any mathematically related awards or scholarships? We welcome your suggestions and recommendations. Please write to dkraines@math.duke.edu.

—David Kraines, DMN Faculty Sponsor

Events

Nobel Laureates To Visit Duke

In spring 2000, two Nobel Laureates in physics will visit the mathematics department.

Val L. Fitch received a Physics Nobel Prize in 1980 for his co-discovery that nature can distinguish at a microscopic level the direction of flow of time (time reversal invariance fails in elementary interactions).

Russell A. Hulse received a Physics Nobel Prize in 1993 for his co-discovery of a binary pulsar system. This system has served as an amazing relativisitic laboratory for testing Einstein's General Theory of Relativity.

Fitch will be at Duke on January 31 and Hulse is scheduled to come on April 4.

DUMU

About DUMU

The Duke University Math Union, or "DUMU" for short, is a club sponsored by the department for undergraduates. Our activities include social events, such as picnics, movies, frisbee games with the Society for Physics Students, and our high-school math contest. Additionally, we invite speakers for the entertaining and informative Undergraduate Lecture Series. If you are interested in hearing about DUMU events and are not already on our mailing list, contact Carl

Miller at `carl@math.duke.edu`. Read on to find out about our current plans. ❧

The High School Math Meet

DUMU is planning to host a contest on Saturday, November 13, for high schools in North Carolina and nearby states, and we need problems and solutions for it. The more people who contribute, the more varied and interesting the contest will be. So, start thinking, and keep you eyes open for intriguing ideas. Math may be a tool and a subject, but it can also be a sport, a game, a kind of art, and thought-provoking fun.

We are looking for original problems of varying difficulty that can be solved in 2-10 minutes using pre-calculus mathematics only. The problems most likely to be used are those that reward insight more than experience. We will also need a few very easy problems. For examples, see last year's contest at `http://www.math.duke.edu/dumu/Problems/`.

Send submissions to Carl Miller at `carl@math.duke.edu`. See `http://www.math.duke.edu/dumu` for more information about the contest. ❧

Competitions

The following contests are open to all undergraduates. They take place on Saturdays and are held in a math classroom in the Physics Building. If you are interested in participating in one of these competitions, or if you would just like more information, contact David Kraines at `dkraines@math.duke.edu`.

- October 30. The Virginia Tech math contest will be held from 8:30 until 11:00 on that Saturday in Math-Physics 120. Nearly 200 students from over 30 colleges and universities in the southeast participate each year. In 1997, eight of the top 16 participants were from Duke. We improved last year with first, second and third places and nine of the top 14 spots.

  For more information, see `http://www.math.vt.edu/events/index.html`.

- December 4. The W. L. Putnam Mathematical Competition is given in two sessions of three hours each. Success in this challenging competition requires ingenuity and mathematical rigor rather than advanced mathematical knowledge. Since 1990, two Duke teams have won the competition and two have finished in second place.

  For more information, see `http://scuish.scu.edu/SCU/Departments/Math/putnam/`.

Department News

Graduation of the Class of '99

Following the university-wide exercises last May, mathematics and physics majors, their families and friends, and a number of mathematics faculty gathered in the Levine Science Research Center Dining Room for a buffet luncheon, followed by a diploma ceremony. Most of our first and second majors reported definite post-graduation plans. At least twelve majors have entered graduate programs, including mathematics, physics, economics, computer science, education, and medicine. A number are working in financial analysis and consulting, and several in software design. ❧

New Faculty

Many professors and research associates have joined the department this fall. Several are associated with the Center for Mathematics and Computation in the Life Sciences and Medicine (CMCLSN).

- **Kayne Arthurs** (PhD Duke), Research Associate, CMCLSN. *Modeling blood flow regulation in the kidney.*

- **Zhuoxin Bi** (PhD U Tulsa), Research Associate. *Statistical inversion techniques for flow in porous media.*

- **Parviz Ghadimi** (PhD Duke) Visiting Assistant Professor. *Microfluidics and thermohydrodynamics lubrication.*

- **Joshua Holden** (PhD Brown) Lecturing Fellow. *Algebraic number theory for number and function fields.*

- **Trachette Jackson** (PhD U Washington) Research Associate, CMCLSM. *Mathematical models in cancer chemotherapy.*

- **Hwanho Kim** (PhD Penn State) Research Associate. *Multigrid methods for elliptic partial differential equations.*

- **Patrick Nelson** (PhD U Washington) Research Associate, CMCLSM. *Dynamics of the human immune system.*

- **David Reed** (D Phil Oxford) Part-time Instructor. *Topology and arithmetic of varieties.*

- **Ramesh Sreekantan** (PhD Chicago) Assistant Research Professor. *Arithmetic of algebraic cycles.*

- **Maxim Vybornov** (PhD Yale) Assistant Research Professor. *Perverse sheaves in stratified spaces and representation theory of quantum groups.*

- **Graham Wilks** (University of Keele) Part-time Instructor. *Boundary-layer heat transfer fluid mechanics.*

Aspinwall Awarded Sloan Fellowship

Assistant Professor of mathematics and physics, Paul S. Aspinwall, has been awarded a Sloan Fellowship. The Sloan Research Fellowships, established in 1955, provide support and recognition to young scientists in order to stimulate fundamental research by young scholars of outstanding promise. Sloan Research Fellows, once chosen, are free to pursue whatever lines of inquiry are of the most compelling interest to them. Twenty-one Sloan Fellows have gone on to win Nobel prizes. Aspinwall plans to use this award to study various aspects of the geometry of superstring theory. Superstring theory is a mathematical model of the universe which has grown out of high-energy particle physics and general relativity in an attempt to construct a "theory of everything". It is highly geometrical in nature which tends to make superstring theory of interest to both mathematicians and physicists. For more information on Sloan fellowships see `http://www.sloan.org/programs/scitech_fellowships.htm.`

Zhou named Fellow of the Guggenheim Memorial Foundation

Associate Professor Xin Zhou has been named Fellow of the John Simon Guggenheim Memorial Foundation for the fall 1999 term. The Guggenheim Fellowship program provides Fellows with blocks of time in which they can work with as much creative freedom as possible. No special conditions attach to them and Fellows may spend their grant funds in any manner they deem necessary to their work. Zhou is continuing his research at the Courant Institute in New York City this fall. For more information, see `http://jsg.gf.org/index.html.`

Leonard Carlitz 1907-1999

Leonard Carlitz, retired James B. Duke Professor of mathematics at Duke, died on September 17 in Pittsburgh. Carlitz was a most prolific mathematician with 770 publications on number theory and related topics appearing in 110 different professional journals. He was on the Duke faculty from 1932-1977. An extended review of the contributions of Professor Carlitz will appear in the next issue of Duke Math News.

News From The Graduate Program

New Graduate Students

- **Shanqin Chen**, Nankai University, Tianjin, China. She will be working with John Trangenstein.

- **Jose Antonio Trujillo Ferreras**, Universidad Autonoma de Madrid, Spain. He is interested in analysis and probability.

- **Gregory Firestone**, University of Washington, Seattle, WA. He is interested in algebraic topology, algebraic geometry, and differential geometry.

- **Angela Gallegos**, New York University. She is interested in mathematical biology.

- **John Greer**, Columbia University.

- **Kevin Kesseler**, Michigan State University, East Lansing, MI. He is interested in applied mathematics and non-linear dynamics.

- **Camelia Lavric**, University of Galati, Romania. She is interested in applied mathematics.

- **Janice McCarthy**, Duke University (MA in biochemistry). She is interested in pure mathematics.

- **Guoqing Yang**, University of North Carolina at Charlotte. He will be working with John Trangenstein.

- **Dan Yasaki**, UNC at Chapel Hill. He is interested in algebra.

Teaching Award

For the first time in its 18 year history, a graduate student has been honored with the L. P. and Barbara Smith Award for teaching excellence for a second time. Laura Taalman was chosen because of her continuing outstanding contributions to teaching and her innovative work in developing course materials. Her students attest, by their words and by their success, to her effectiveness. One can look at this year's coursepack for Math 25L/26L to see some of the fine course development that she has done. Indeed, the combination of her teaching skill and her influence on our course development over a number of years have earned her this unprecedented recognition.

Taalman recently received the *Dean's Award for Excellence in Teaching*. She was nominated for this $1250 award by her students and selected by the Graduate School, Trinity College, and Alumni Affairs.

The Smith teaching award was made possible by a generous donation from Captain L. P. Smith and Barbara Smith, who established the Smith Award in 1981. Captain Smith was Supervisor of Freshman Instruction in the Mathematics Department from 1973 until his retirement in 1982. The Smiths' goal was to reward those graduate students for their efforts to become fine teachers. The Smiths are now enjoying their retirement in the Seattle, Washington, area.

Food for Thought on Fridays

Each week, the math graduate students host an informal talk in which graduate students and faculty get a chance to share their ongoing research or some mathematical gem they have discovered. These talks are held every Friday afternoon at 4:00 in Room 120 Physics, preceded by tea at 3:30. Check the posted math calendar or `http://math.duke.edu/cgi-bin/mcal` for the next speaker and title. No previous knowledge is assumed other than a solid undergraduate math experience; everyone is encouraged to attend.

Opportunities

Goldwater Scholarship

The prestigious Goldwater Scholarship is intended for sophomores and juniors planning careers in mathematics, engineering, and the natural sciences. It is worth up to $7,500 annually, covering tuition, fees, books, and room and board. Sixteen math majors at Duke have won this award since 1989.

Preliminary applications are available in Allen Building 04 and are due by Monday, October 18. For more information, please see the Goldwater web site `http://www.act.org/goldwater` or Dean Mary Nijhout in room 04 of the Allen Building.

*C*hance favors only the prepared mind.

—Louis Pasteur

Office Assistant

The Math Department needs student office assistants. Flexible work schedules can be arranged. Interested students should fill out an application in room 121 or see Mrs. Wilkerson in 121E.

Alumni News

This fall, the Duke Math News is being sent to many former math majors. Please share news about your life or career with others in the Duke mathematical community. If you are in contact with other former math majors, let them know that we would like to put them on the mailing list as well. You can always read about Duke math at `http://www.math.duke.edu`, but we want to read about you. Send information or inquiries to `dkraines@math.duke.edu`.

Notes from the DUS

As the Director of Undergraduate Studies in the Mathematics Department, I want to welcome you all back! We have a very lively environment here for learning mathematics, and I will be looking for ways to make it better. I will be glad to have imaginative suggestions.

If you declared your major or minor in math before the school year started, you should have received the new edition of our *Handbook for Mathematics Majors and Minors*. If you have not received one, you can pick up a copy in the Mathematics Office, Room 121, from a shelf on the right as you enter the office. The *Handbook* is also available at the department's web site (`http://www.math.duke.edu`), under the heading "The Undergraduate Program." Lots of other information can be found at the web site.

Registration for the spring semester begins on Wednesday, October 27. If your first major is math, you will have an advisor in the Mathematics Department. For those of you who recently declared a first major in math, I will soon be assigning you an advisor. If you are not a first major in math, your advisor is in another department, but you are welcome to talk to me about the math program if you wish.

Mathematics courses suitable for majors and minors (past 104) and scheduled to be offered in the spring semester include:

MTH 114, 121, 126, 128S, 131 (three sections), 133, 135 (two sections), 136 (two sections), 139, 160, 197S, 201, 204, 206, and beyond.

Math 197S will be a seminar in set theory, taught by Professor Hodel. Math 128S will be a seminar in number theory taught by Professor Holden. Math 160, Mathematical Numerical Analysis, has been revised by Professor Witelski, a dynamic young applied mathematician who joined our department last year. Details about several courses will be given in the course synopses.

Have a great semester!

         —Tom Beale, beale@math.duke.edu

Problem Corner

Solutions from Last Issue

Problem 1: *An End to Charity*

In a group of n people, each person possesses an initial amount of money measured in whole dollars. The group makes a series of two-person transfers under the requirements that: (1) transfers are made only in whole dollars, and (2) each transfer must leave the giver with more money than the taker. Show that only a finite number of such transfers can be made.

Solution to Problem 1:

Let d_i represent the dollar value possessed by person i. Consider the sum $\sum_{k=1}^{n} d_i^2$. If person i gives k dollars to person j, we are given that $d_i - d_j > k$ initially; thus

$$
\begin{aligned}
k - d_i + d_j &< 0 \\
2k(k - d_i + d_j) &< 0 \\
d_i^2 + d_j^2 + 2k(k - d_i + d_j) &< d_i^2 + d_j^2 \\
(d_i^2 - 2d_i k + k^2) + (d_j^2 - 2d_j k + k^2) &< d_i^2 + d_j^2 \\
(d_i - k)^2 + (d_j + k)^2 &< d_i^2 + d_j^2
\end{aligned}
$$

Therefore the sum $\sum_{k=1}^{n} d_i^2$ decreases every time a transfer is made. Since the sum can only take on nonnegative integer values, it follows that only a finite number of transfers can be made.

Problem 2: *Slippery Subsets*

S is a set of 2000 elements, and P is a set of subsets of S such that if A and B are in P, A is not a proper subset of B. Find the maximum number of elements that P can contain.

Solution to Problem 2:

For any subset T of S with k elements, let $C(T)$ be the set of all sequences $(s_1, \ldots, s_{2000})$ in which the elements of T are given in no particular order by the first k terms $(s_1, \ldots, s_k)$, and the remaining elements $S - T$ are given in no particular order by the last $2000 - k$ terms $(s_{k+1}, \ldots, s_{2000})$. There are k! possible orderings of the elements of T and $(2000 - k)!$ possible orderings of the elements of $S - T$, therefore there are exactly $k!(2000-k)!$ such sequences.

Suppose T and U are contained in P, and the sequence $(r_1, \ldots, r_{2000})$ is contained in both $C(T)$ and $C(U)$. Assume without loss of generality that $|T| \leq |U|$; then $T = \{r_1, \ldots, r_{|T|}\}$ is a subset of $S = \{r_1, \ldots, r_{|S|}\}$ — this is contradiction, since we assumed no set in P could be a subset of another set in P. It follows that the collection $\{C(T) : T \in P\}$ is pairwise disjoint.

The minimum value of $k!(2000 - k)!$ over all k is $1000!^2$ (an exercise for the reader). And finally, there are a total of 2000! ways to arrange the elements of S in an arbitrary sequence. Therefore,

$$
\begin{aligned}
2000! = |C(S)| \quad &\geq \quad \left| \bigcup_{T \in P} C(T) \right| \\
&= \quad \sum_{T \in P} |C(T)| \ \text{(by disjointness)} \\
&\geq \quad \sum_{T \in P} 1000!^2 \\
&= \quad |P| 1000^2
\end{aligned}
$$

Therefore $|P| \leq \frac{2000!}{1000!^2} = \binom{2000}{1000}$. This maximum is acheived if we let P be the set of all 1000-element subsets of S. Thus the answer is $\binom{2000}{1000}$.

New Problems

Problem 1: *Fibonacci Sum*

Let $F_n = (1, 1, 2, 3, 5, \ldots)$ be the Fibonacci sequence (defined by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$). Compute $\sum_{k=1}^{\infty} \frac{F_n}{2^n}$.

Problem 2: *Massive Matrix*

Let A be a 2000×1998 matrix with real entries. Show that AA^{T} contains at least one nonnegative entry which is not on the main diagonal.