Duke University Math News

September 29, 2004

Welcome

As the Director of Undergraduate Studies in Mathematics, I would like to welcome you back to classes. I hope that you had a productive, or an interesting, or a fun summer (for myself, I had all three). We have a very lively environment at Duke for learning mathematics; even so, I welcome any suggestions that you may have to make it even better.

The 2004-2005 edition of the *Handbook for Mathematics Majors and Minors* is now available, and you can pick up a copy in the Mathematics Office, 121 Physics. The Handbook is also available at the web site of the department `http://www.math.duke.edu` under the heading The Undergraduate Program.

Registration for the spring semester begins on Wednesday, October 27; advising begins on Monday, October 25. If your first major is mathematics, you will have an advisor in the Mathematics Department. For those of you who recently declared a first major in mathematics, I will assign you an advisor soon. If you are not a first major, your advisor will be in the department of your first major, but please feel free to talk to me about your selection of mathematics courses and the requirements for the mathematics major. `hodel@math.duke.edu`.

Here is a preview of courses above MTH 104 that will be offered next spring: 111 (one sections), 114 (three sections), 121, 126, 128S, 131 (two sections), 132S, 133, 135 (two sections), 136, 139, 188, 196S, 201, 204, 206, and numerous other 200 level courses.

—*Richard Hodel*
Director of Undergraduate Studies

Undergraduate News

Math Union

The Duke University Math Union organizes activities of interest to math majors and other mathematically inclined students including picnics, ice cream socials, games nights, and frisbee contests. In addition, DUMU helps organize the major events and opportunities for undergraduates that you can read about below. To get on the mailing list for announcements of future events, send a note to Oaz Nir at `on5@duke.edu`.

High School Math Meet

This ARML style meet will be held on Saturday, October 30. Last year, over 120 high school students from Georgia to Virginia came to Duke for this fun event. Duke students are needed to make up contest problems, to help with registration, to monitor the sessions, and to grade the solutions. Those who went to high school in the Southeast should contact their math teachers there and encourage them to send a team. Contact Paul Wrayno at `pmw5@duke.edu` for more information.

Competitions

The following competitions are open to all regularly registered undergraduates. There is no cost, lunch is provided and monetary prizes are awarded. If you are interested in participating or if you would just like more information, contact David Kraines at `dkraines@math.duke.edu`.

The Virginia Tech Regional Math Contest will be held from 9:00 to 11:30 on Saturday, October

23. The W.L. Putnam Mathematical Competition will be held on Saturday, December 4, from 10:00 to 1:00 and 3:00 to 6:00.

The Problem Solving Seminar/Math 149S is a half credit course that meets one day a week during the fall term. Students work on challenging mathematics problems in the company of others with similar interests. The seminar meets Tuesdays from 9:00 pm to 11:00 pm in Physics 120. Students interested in math competitons are welcome to drop in.

PRUV Fellows

Through a grant from the National Science Foundation VIGRE program, up to eight current sophomores and juniors may be supported for six weeks of summer research in mathematics and its applications. For more information on this PRUV Fellow program, see `http://www.math.duke.edu/vigre/pruv/`.

Undergraduate Math Lectures

DUMU plans to host one or two lectures this year by distinguished professors who will speak on mathematical topics of general interest. Professor Jordan Ellenberg of Princeton University is tentatively schedule to come in early spring 2005. Please forward suggestions for other speakers to Oaz Nir `on5@duke.edu`

Undergraduate Courses

Engineering Math

In response to a new mathematics requirement of the Pratt School of Engineering, the department of mathematics has developed the following course sequence:

> **Math 107** *Linear Algebra and Differential Equations.* Systems of linear equations, matrix operations, vector spaces, linear transformations, orthogonality, determinants, eigenvalues and eigenvectors, diagonalization, linear differential equations and systems with constant coefficients, applications, computer simulations. Intended primarily for engineering and science students. Prerequisite: Mathematics 103. Not open to students who have had Mathematics 104 or 111.
>
> **Math 108** *Ordinary and Partial Differential Equations.* First and second order ordinary differential equations with applications, Laplace transforms, series solutions and qualitative behavior, Fourier series, partial differential equations, boundary value problems, Sturm-Liouville theory. Intended primarily for engineering and science students. Prerequisite: Mathematics 107. Not open to students who have had either Mathematics 111 or 131.

Both Math 107 and 108 count towards the minor in mathematics. Math 107 cannot be counted as one of the courses above 104 toward an A.B. or B.S. degree in mathematics. Math 104 and math 131 remain the approriate choices for mathematics majors.

Mathematical Model Seminar

In Math 196S this spring, students will study population dynamics and language learning. Mathematical topics will include dynamical systems, bifurcations, Bayesian inference, and Markov chains. Topics from linguistics will include some basic theories of syntax, morphology, and phonology, along with historical topics such as the evolution of English word order. The overall goal is to address the question of why languages change, while developing mathematical modeling tools useful in a variety of disciplines in the natural and social sciences. Students will complete several projects over the course of the semester. Prerequisite: Math 111, 108, or 131 or permission of the instructor. Math 135 is recommended but not required. Instructor: W. Garrett Mitchener

Graduate Program News

New Graduate Students

The following students have entered the graduate program in mathematics this fall. Previous educational institutions and advanced degrees are listed after their names.

- **Shu Dai,** Masters in Physics and BS in Physics, University of Science and Technology of China
- **Conrad Hengesbach,** BS in Mathematics, University of Warwick, Conventry, UK
- **Hyeongkwan Kim,** BS in Physics, Pohang University, Korea, Masters in Physics Michigan State University
- **Anthony Narkawicz,** BS in Mathematics, Virginia Tech
- **Andrea Watkins,** BS in Mathematics, Howard University
- **Jason Wilson,** BS in Computer Science, Rose-Hulman Institute, Masters in Mathematics, California State University and a Masters in Computer Science, UNC-Chapel Hill

Anthony Narkawicz brings an NSF graduate fellowship and Andrea Watkins brings a 6-year National Physical Sciences Consortium fellowship.

No human investigation can be called real science if it cannot be demonstrated mathematically

—Leonardo da Vinci

Teaching Award

At the first meeting of the Department this year, Professor Dick Hain presented the annual L. P. Smith and Barbara Smith Award for Teaching Excellence. The three graduate students recognized with this honor are Tim Lucas, John Cain, and Dave Anderson.

This Award is given to graduate students who have demonstrated their commitment and success in teaching over a long period of time. Tim, John, and Dave have been praised by their students for their work in the classroom, for their willingness and ability to help students outside class, and for inspiring students to study mathematics. These teachers have also made contributions to the department's teaching in other ways, such as helping with curriculum development, assisting with new teacher training, and teaching special courses that are more difficult to deliver than the usual course.

This award not only conveys public recognition of the outstanding teaching of these graduate students, but there is also a substantial fellowship that goes to the recipients. This year's winners were each presented a fellowship of $2000. This fellowship was made possible by a generous donation from Captain L. P. Smith and Barbara Smith, who established a fund in 1981 for this purpose.

Following his retirement from the U.S. Navy, Captain Smith became Supervisor of First-year Instruction in the Mathematics Department in 1973, and continued until his (second) retirement in 1982. The Smiths' goal was to reward those graduate students who work hard to become fine teachers. The Department not only thanks these young teachers for their contributions to our teaching program, but we are also thankful to the Smiths for their generosity and for providing the Department a means of recognizing the outstanding achievements in teaching of these graduate students.

- Lewis Blake

It is impossible to be a mathematician without being a poet in soul

—Sophia Kovalevskaya

Faculty News

New Faculty

The following professors and research associates have joined the department this fall.

- **Bakhtin, Yuri** (Moscow State University), Visiting Assistant Professor. *Probability and Stochastic Process*.
- **Hubert Bray** (Stanford University), Professor. *Differential Geometry*.
- **Clark Bray** (Stanford University), Lecturing Fellow. *Algebraic Topology*.
- **Jane Hawkins** (University of Warwick), Visiting Professor. *Ergodic theory and dynamical systems*.
- **Anandhan Jayaraman** (University of Pittsburgh), Research Associate. *Partial differential equations, scientific computing*.
- **Eric Katz** (Stanford University), Assistant Research Professor. *Algebraic Geometry*
- **Anita Layton** (University of Toronto) Assistant Research Professor. *Mathematical modeling of renal physiology*.
- **Adam McInroy** (Columbia University), Visiting Assistant Professor. *Algebraic geometry*.
- **Ruriko Yoshida** (University of California at Davis) Assistant Research Professor. *Applied math*

Faculty News

The department has the great fortune to add brothers Hubert and Clark Bray to its faculty. In 1918, their grandfather Hubert received the first doctorate awarded from Rice University and taught mathematics there until he retired. Hugh and Clark continued the family tradition as undergraduates at Rice. Each in turn received his doctorate in mathematics from Stanford University.

Before coming to Duke as professor of mathematics, Hubert Bray taught at MIT and Columbia. He studies Differential Geometry, especially problems motivated by Physics and General Relativity. In 1973, Roger Penrose conjectured a relationship between the mass of black holes and the total mass of the spacetime. Bray proved an important case of this Penrose Inequality by defining a new flow of metrics which preserves nonnegative scalar curvature. He continues his research with other significant results in this area. This August, Hugh and his wife Heidi enjoyed their honeymoon in the NC mountains.

After following his older brother through college and graduate school, Clark taught at Santa Clara University. A year later, he was invited to return to Stanford to help manage their freshmen and sophomore math program. As Lecturing Fellow in mathematics at Duke, he expects to be actively involved in all aspects of undergraduate math education.

The brothers are happy to be reunited here and look forward to getting to know the students and faculty at Duke.

Communicating Mathematics

by Clark Bray

I feel that one of the most important lessons I learned as an undergraduate math major is that mathematics is not only a powerful science, but also an elegant and beautiful art. This perspective has been at the foundation of the great enjoyment and personal satisfaction I have found in mathematics over the years, and in large part motivated me to pursue mathematics as a career.

As a teacher, much of the satisfaction that I derive from my work comes from the extent to which I am able to communicate this specific perspective to my students. Some may be motivated, like I was, to choose mathematics as a career. For the majority that do not, the hope is that they will nevertheless retain an appreciation for the beauty of the rigor, elegance, and natural simplicity in mathematics.

I am thrilled to become a part of the mathematics community at Duke, and look forward to being able to make contributions to the department

Reminiscences of Undergraduate Life

by Hubert Bray

When I was an undergraduate at Rice, I was very interested in the prime number theorem. The prime number theorem, first conjectured by Gauss, states that if $\pi(x)$ is the number of primes less than or equal to x, then

$$\lim_{x\to\infty} \frac{\pi(x)\ln(x)}{x} = 1.$$

This turns out to be equivalent to the following statement. Let $\Lambda(x)$ be sum of $\phi(k)$ for all $1 < k \leq x$, where $\phi(k) = \ln(p)$ whenever $k = p^n$ for some prime p and some positive integer n. Since powers of primes are even more sparse than primes, $\Lambda(x)$ is roughly the sum of the natural logarithms of all of the primes less than or equal to x. Then a common version of the prime number theorem is that

$$\lim_{x\to\infty} \frac{\Lambda(x)}{x} = 1.$$

Notice that both versions of the prime number theorem can be interpreted as saying that the density of the prime numbers is asymptotic to

$$\frac{1}{\ln(x)}$$

in some sense.

This was one of many problems I was fascinated by as an undergraduate. I used to spend a lot of time going from professor to professor at Rice asking about problems like this one. Most of the problems were not homework assignments but questions I was just curious about.

Usually the first professor, after telling me what he knew about the question, would refer me to another professor who was more of an expert. After talking to several professors, I was able to make some progress on the problem. Since I had the benefit of the input from these professors, the next professor I visited would be impressed with my knowledge on the subject and so more willing to help.

Although I did not find my own proof of the prime number theorem, after focusing on the problem very hard for several months I did duplicate a result of Chebyshev, which states that $\frac{\Lambda(x)}{x}$ is bounded above and below by constants. Looking back, this is where I began doing research.

Since Duke has the same open atmosphere as Rice, I encourage Duke students, both undergraduate and graduate, to take advantage of professors' open door policies to talk about any math problems of interest.

Students should strike a balance between simply learning more mathematical facts and practicing creative thinking. Homework problems may suggest other questions. Tackling such a problem, whether or not a complete solution is ultimately found, is one of the greatest joys for a mathematician!

Major Grants

Professor Petters was recently awarded a National Science Foundation grant from the program *Mathematical Sciences: Innovations at the Interface with the Sciences and Engineering*. He is Principal Investigator of the $600,011 grant, which is for over three years and funds a research collaboration between Duke and MIT. The co-Principal Investigators Paul Schechter (astronomer, MIT) and Brian Rider(probabilist, University of Colorado, Boulder) will work closely with Professor Petters as part of a research team that includes scientists from Harvard, Postdam University in Germany and Rutgers University. Professor Petters will be responsible for guiding the team's research, which is at the interface of mathematics and astronomy. In particular, the team will explore the mathematics of stochastic gravitational lensing with application to dark matter and quasar flux ratio anomalies.

Assistant research professor Anita Layton has received an ADVANCE Fellows Award worth $435,285; *Mathematical modeling of renal physiology*. The aim of the project is to gain a better understanding of the mechanism by which mammals produce concentrated urine. She expects to disseminate modeling methodologies and computer visualization techniques to aid other experimentalists to better understand the significance of much new experimental data now emerging through techniques involving computer-assisted micro-anatomy and transgenic animals.

Distinguished Teaching Award

Visiting assistant professor Parviz Ghadimi was named the winner of the 2004 Alumni Distinguished Undergraduate Teaching Award. He will be recognized at Founder's Day on September 30 with plaques and a stipend of $5000.

Each year, undergraduate students are encouraged to nominate their favorite professors for this award. A committee of ten undergraduates recommends one of the 40 or more nominees to the Board of Directors of the Duke Alumni Association which makes the final decision. Ghadimi has been nominated four times in recent years. He is the first faculty member in mathematics at Duke to have won the Alumni Teaching Award.

Ghadimi received his doctorate in Mechanical Engineering from Duke. He has held the positions of research professor and adjunct professor in that department and is also an adjunct professor at NC State University and UNC. He has been teaching in the Duke mathematics department since 1999.

Ghadimi states "I am humbled and honored to have been nominated by my students for this prestigious award. It is indeed wonderful to feel rewarded and fulfilled and to see that my efforts have been valued. The award would not have been possible without the support and cooperation of my great colleagues and the wonderful staff of the Duke Mathematics Department."

Problem Corner

Solutions from Last Issue

1. *Problem*. We will call an integer number n "good" if it can be presented in the form $n = x^2 + y^2$, where x and y are integers. Show that if a and b are "good", then their product ab is also a "good" number.

   Solution. The fact follows directly from the identity $(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 = (ad - bc)^2$.

2. *Problem*. Find all natural numbers n such that $n|2^n - 1$ (Balkan Mathematical Olympiad).

   Solution. Let $n > 1$. Since $n|2^n - 1$, n should be an odd number. Let p be its smallest prime divisor. From Fermat's Theorem, $2^{p-1} \equiv 1 \pmod{p}$, and from the problem statement we have that $2^n \equiv 1 \pmod{p}$. Let s be the smallest number such that $2^s \equiv 1 \pmod{p}$. We can present n in the form $n = sq + t$, where $0 \leq t < s$. Then $2^n \equiv 2^{sq}2^t \equiv 2^t \equiv 1 \pmod{p}$ and therefore, since s is minimal, t must be zero. From here, $s|n$ and by analogy $s|p - 1$. But then s is a divisor of n smaller than p, from where s should be 1, which means that $2 \equiv 1 \pmod{p}$, which is impossible. Therefore, only $n = 1$ can be a solution. It is easy to verify that it actually is.

New Problems

1. **Problem 1** Prove that if $7|a^2 + b^2$, then $7|a$ and $7|b$.

2. **Problem 2.** Let a, b, m and n be natural numbers and a and b be relatively prime. Show that $(a^n - b^n)|(a^m - b^m)$ if and only if $n|m$.

3. **Problem 3.** Show that the polynomial

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}$$

is irreducible over $\mathbb{Q}[x]$ for every natural number n.

Submit solutions or suggestions for new problems to Problem Editor Nikifor Bliznashki nb22@duke.edu

Each problem that I solved became a rule which served afterwards to solve other problems

—Rene Descartes

Duke Math News

The *Duke Math News* is published several times a year and is distributed to those in the Duke mathematics community by campus mail. For previous editions and other news, see www.math.duke.edu/news/. We welcome items of interest for our next issue. Send them to jones@math.duke.edu or dkrain@duke.edu

To read about other news, honors and events concerning mathematics at Duke, visit www.math.duke.edu/news/. The on-line calendar at www.math.duke.edu/mcal lists both regular and special seminars and colloquia for the upcoming weeks. The department maintains video archives of talks, lecture series and special conferences at Duke, many of which are available, on-line. See www.math.duke.edu/computing/broadcast.html for more information.

—David Kraines, DMN Faculty Sponsor

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