Duke University
Math News
April 23, 1999

Math Department Celebration

The annual department party, held in the math department lounge on Wednesday, April 21, brought together mathematics majors and minors, graduate students and their professors and deans to chat with each other while sharing a picnic supper. This event allowed the students, faculty and administrators to recognize the accomplishments of the many prize, award and scholarship winners.

President Nannerl Keohane honored Andrew Dittmer at the ceremony for his many accomplishments. In his first year at Duke, Dittmer published a research paper in the American Mathematics Monthly on an n dimensional generalization of the vector product. After two years of service in Italy for the Mormon Church, Dittmer returned to his studies at Duke fluent in several Italian dialects and ready to work on a sophisticated mathematical research problem. Under the direction of Professor Richard Hain, Dittmer wrote his undergraduate thesis generalizing classical formulas of Heron of Alexandria and Brahmagupta on the area of a triangle and a quadrilateral inscribed in a circle in terms of the side lengths. Dittmer applied Galois Theory to show that, for polygons with more than 4 sides inscribed in a circle, the radius of this circumscribing circle does not, in general, lie in the field of surds generated by the side lengths. In particular, there can be no general ruler and compass construction of this circle.

Dittmer was a member of both of Duke's championship Putnam teams as well as one of Duke's two second place teams. Although not a computer science major, Dittmer contributed to Duke's 5th place finish, the highest among the 22 US teams competing, in the international collegiate programming contest held in the Netherlands in April 1999. Dittmer, an AB Duke Scholar and a Faculty Scholar, received his bachelor's degree in December 1998 with highest distinction and is currently a graduate student in mathematics at Duke.

President Keohane retired Dittmer's jersey with team number $\mathfrak{S}_7$, the symmetric group of 7 letters, and this jersey will hang along with that of Jeff Vanderkam '94 in the math department lounge.

Graduation luncheon

Graduating students who major in mathematics are cordially invited to attend a luncheon in the Levine Science Research Center Dining Room after Graduation Exercises on Sunday, May 16. In a brief ceremony at the end of the luncheon, Professor J. Thomas Beale, Director of Undergraduate Studies will present the diplomas to those with math as their first major.

All great theorems were discovered after midnight.

—Adrian Mathesis

Contests

Modelers Outstanding Again

In early February, Duke's team for the Mathematical Contest in Modeling spent a grueling weekend solving a problem in applied math. This year's team, consisting of Sam Malone '02, John Thacker '01, and Garrett Mitchener '99, chose to work on the room problem, which required them to develop ways to determine the maximum legal occupancy for a room. Their paper earned a ranking of Outstanding, the highest possible award. The team members will each

receive a cash prize from INFORMS, the Institute for Operations Research and Management Science. This is the second straight Outstanding designation for Duke and for Mitchener and Thacker.

The other problems for this year were to predict what would happen if a large meteorite were to hit the south pole, and to analyze a set of ground water data. Unlike the problems in most math competitions, these problems have no single right answer. Solution papers are judged based on creativity, clarity, and completeness.

Two Duke students named Putnam Fellows

Jonathan (Nathan) Curtis '01 and Kevin Lacker '02, were named Putnam Fellows for ranking among the top 5 of the 2581 contestants from 419 colleges and universities in Canada and the United States in the 59th annual William Lowell Putnam Mathematical Competition held December 5, 1998. Curtis and Lacker will each receive $2500 for their most impressive scores of above 100 on this prestigious six-hour event. This year, the median score was a relatively high 9 [sic] out of 120. John Clyde '01 and Carl Miller '01 earned Honorable Mention for ranking within the top 2% of the best math students in North America. Andrew Dittmer placed among the top 100, Michael Colsher '01, Sarah Dean '00, and Spencer Shepard '00 each ranked among the top 5% with John Hyde '99, Daniel Neill '01, and Jeffrey Mermin '00 among the top eighth. Frank Caruso '02 and Andrew Goss '02 were among the top quarter of this elite group. The team of Curtis, Dittmer and Miller finished 6th in the nation. Duke teams finished first or second four times since 1990. Two Duke students had been Putnam Fellows in the past: Jeff Vanderkam '94 in 1992 and Craig Gentry '95 in 1993.

ACM International Computer Programming Contest

Following their regional victory, the Duke ACM Programming Team of John Clyde, Andrew Dittmer, and Meetesh Karia placed fifth in the world at the 23rd Annual ACM International Collegiate Programming Contest. The competition was held on April 8–12 at Eindhoven University of Technology in the Netherlands. The contestants were asked to solve eight challenging programming problems in five hours. Overall, 1,457 teams from around the world participated in the competition, 62 of which advanced to the world finals. For more information, see `http://acm.baylor.edu/acmicpc/`.

Awards and Scholarships

Julia Dale Prize Winners

The 1999 Julia Dale Prize for excellence in undergraduate mathematics has been awarded to seniors Christopher Beasley, Johanna Miller and Garrett Mitchener. Julia Dale was an assistant professor of mathematics at Duke University who died in 1936 quite early in her career. Shortly thereafter, friends and relatives established a prize in her name to be awarded to the most distinguished mathematics students at Duke. For more information on Professor Dale and for a list of previous winners, see `http://www.math.duke.edu/undergraduate/Dale/dale.html`.

Beasley, a mathematics and physics major, has written his senior thesis on super string theory under the supervision of Ronen Plesser of the Center for Geometry and Theoretical Physics. His thesis studies some examples of the "holographic" correspondence between four-dimensional quantum field theory and five-dimensional theories of gravity. Beasley, an AB Duke Scholar, a Goldwater Scholar and a Faculty Scholar, will study mathematical physics at Cambridge University next year as a Churchill Scholar and then plans to get his doctorate at an American university with the support of a National Science Foundation graduate fellowship.

Miller is a double mathematics and chemistry major with a minor in music who has opted for a career in physics. In her first year at Duke, she won the Karl Menger award for receiving the highest score among all Duke students in the W. L. Putnam Competition. That summer, she

wrote a paper at an REU program that won a prize for one of the best papers by an undergraduate to be published in the Pi Mu Epsilon mathematics journal. Miller, an AB Duke Scholar and a Goldwater Scholar, has been awarded a National Science Foundation graduate fellowship to study physics at the University of Chicago.

Mitchener, a mathematics and computer science major, has been editor of the Duke Math News since his first year at Duke and has led DUMU, the math club, for the past two years. Starting in his senior year at the North Carolina School of Science and Mathematics, Mitchener contributed to three Math Modeling teams that have been designated as Outstanding and two that have been ranked Meritorious. He presented the results of his senior thesis to the mathematics department on April 21, immediately preceding the Math Celebration. In this work, Mitchener studied lattice sphere packings in n dimensional euclidean space. Mitchener is a NC Math Scholar and a Goldwater Scholar. He plans to attend graduate school in applied math at Princeton University.

Faculty Scholar

Sarah Dean '00 has been named Faculty Scholar for 1999–2000. This, the highest honor awarded by Duke faculty to our undergraduates, is given to the 3 or 4 rising seniors who have best demonstrated significant achievement in independent scholarship. Since 1991, seven math majors have received this prestigious award.

Karl Menger Award

The Karl Menger Award is given in recognition of superior performance in the William Lowell Putnam Mathematical Competition. This year's awards go to Putnam Fellows, Nathan Curtis '01 and Kevin Lacker '02 and to John Clyde '01 for his rank of 28 among the 2581 contestants. This is the second Menger award for Curtis. Karl Menger, an influential twentieth century mathematician from Vienna, held academic positions in Europe and the United States. The Karl Menger Award was established by a gift to Duke University from George and Eva Menger-Hammond. Karl Menger was Eva Menger-Hammond's father.

Goldwater Scholarships

Michael Colsher '01 and Sarah Dean '00 were among 304 undergraduates in the country, including just 17 math majors, to be named 1999 Barry M. Goldwater Scholars. This national scholarship rewards outstanding ability in mathematics, natural science and engineering with up to $7500 per year towards college expenses. Since its establishment in 1989, 16 mathematics majors and 21 other Duke undergraduates have been among the winners. See `http://www.act.org/goldwater` for more information.

Graduate Student News

First year graduate student, Michael Silverstein, was one of 45 students in the US to receive an NSF graduate fellowship in mathematics this year.

Yiannis Vlassopoulos received his doctorate in December for his thesis "Quantum Cohomology and the Loop Space." Liya Zhornitskaya and Ben McKay received their doctorates this spring. Zhornitskaya's thesis was "Positivity preserving numerical schemes for lubrication type equations," McKay's thesis was "Duality and Itegrable Systems of Pseudoholomorphic Curves." McKay will be a Postdoctoral Fellow at the Max Planck Institut für Mathematik in Bonn, Germany, and Zhornitskaya will be a Postdoctoral Fellow at the Institut für angewandte Mathematik der Universität Bonn.

Geometry and Theoretical Physics at Duke

An interdisciplinary program linking, literally and figuratively, the math and physics departments at Duke is now in full gear. David Morrison, James B. Duke professor of mathematics with a secondary appointment in physics, founded this program to bring together mathematicians and physicists to study string theory. Strings are thought to be exceptionally small

objects moving in 10 or 11 dimensional space and many physicists and mathematicians believe that they determine the fundamental structure of the universe. Assistant professors Paul Aspinwall and Ronen Plesser as well as post-docs and graduate students participate in this "collaborative effort between mathematicians and physicists to try to understand this theory." The work of this group has proved useful to both the fields of mathematics and physics, with insights into the relationships between general relativity and quantum mechanics as well as in algebraic and differential geometry. For more information, see `http://www.cgtp.duke.edu`.

Lectures

The Elegant Universe

Last March, Brian Greene, professor of mathematics and physics at Columbia University who also holds adjunct appointments at Duke and Cornell Universities, enthralled a large audience at Duke University by describing how recent results in string theory may help researchers realize Einstein's dream of unifying general relativity and quantum mechanics. His talk used computer graphics and a clip from the Star Trek series to convey to the overflow audience, ranging from freshmen and research professors, some of the fascinating aspects of what some have called the Theory of Everything. Since the fall of 1997, Greene has taught a course in Quantum Field Theory for students and faculty at Duke, Columbia and Cornell Universities via a remote video hookup.

I am coming more and more to the conviction that the necessity of our geometry cannot be demonstrated, at least neither by, nor for, the human intellect...geometry should be ranked, not with arithmetic, which is purely aprioristic, but with mechanics.

—Karl Friedrich Gauss

Other news

Summer research plans

Several undergraduates will use mathematics in their summer jobs. Three have been accepted at REU (Research Experiences for Undergraduate) programs around the country. Kevin Lacker '02 will attend a program in differential geometry at Tulane University, Sam Malone '02 will study matrix theory at William and Mary College and John Thacker '01 will be the fifth Duke student since 1994 to attend the REU program at University of Washington. These highly competitive programs offer six to eight students the opportunity to do mathematical research under a faculty mentor. Stipends paid by the National Science Foundation range from $2000 to $3000 for these six or seven week programs. Sarah Dean '00 will work for the National Security Agency and British Intelligence outside London this summer. Jeff Mermin '00 will join the Director's Summer Program of the National Security Agency. Johanna Miller '99 will be a TA/counsellor at the Pennsylvania Governor's School for the Sciences, a five week program for rising high school seniors. Carl Miller '01 will be a counselor at the Ross Young Scholars Program at Ohio State.

Duke math shirts

The 1998–1999 Duke Math T shirts are available in room 121 for $8 for L and XL and $9 for XXL while supplies last. The design is based on the 1998 Putnam problem: A right circular cone has base radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?

Math Horizons

Several issues of this informative journal are available first come-first serve in room 121. The February 1999 issue reviews the full story of the attempt to legislate the value of π and a dozen "proofs" that $1 = 2$. Math humor comes alive in the interview with professor/professional comedian Lew Lofton.

Math Degree Candidates, Academic Year 1998-99

First Majors

Edward Fedrico Babbage
Alexander Sabo Brodie
Wen Hsin Chang
Rani Larissa Croager
Jeffrey Peter Dilisi
Andrew Olstrom Dittmer
Garrett Ryan Greeby
Rachel Haber
Mark Bryant Hall
Eric Richard Hwang
John Charles Hyde, III
Aparna Prakahs Keshaviah
Aaron David Kuebler
Trajan Shaka Langdon
Andrew White Lathren
Martine Elizabeth Lellis
Christopher Anthony Maliaro
Michael Thomas Manuel
David Michael McKinley
Robert Raymond McNerney, Jr
Johanna Lynn Miller
William Garrett Mitchener
John LeRoy Parker, Jr
Rodney Peele
Aqueelah Nafeesa Rashada
Owen William Roberts
Glen Alan Toomayan
Paul Russell Womble

Second Majors

Christopher Edward Beasley
Geoffrey Carlton Berry
Charles Alan Bester
Landon Prentice Cox
David Tien Huang
Mark Gilbert Jackson
Meetesh Mahendra Karia
Dharmen Dhiren Mehta
Jeff Thomas Pavlovic
Robert Earl Ross
Joel Brian Watkins

Minors

John David Cann
Asha Takiya Jones
Ka Po Kwan
Andrew Gregory Moore
Rachel Aliza Polimeni
Matthew Christopher Risley

Master of Arts

David Ambrose
Anne Collins
Craig Grabowski
Christopher Hale
Marianty Ionel
Rakesh Kariholoo
Darren Oldson
Andreas Toumassis

Ph.D.

Benjamin McKay
Yiannis Vlassopoulos
Liya Zhornitskaya

Problem Corner

Solutions from Last Issue

Solution to Problem 1:

$f(x) = x + f(x^2)$, thus $f(x^2) = x^2 + f(x^4)$. Substituting the second equation into the first yields $f(x) = x + x^2 + f(x^4)$. Similarly, $f(x) = x + x^2 + x^4 + f(x^8)$. Continuing inductively, we have $f(x) = \sum_{k=0}^{n-1} x^{2^k} + f(x^{2^k})$ for any n. Taking the limit as n approaches infinity, $f(x) = \sum_{k=0}^{n-1} x^{2^k} + \lim_{n\to\infty} f(x^{2^n})$. x^{2^n} approaches zero as n approaches infinity, therefore by continuity this becomes $f(x) = \sum_{k=0}^{n-1} x^{2^k} + f(0)$. Differentiating this power series yields $f^{(63)}(0) = 0$ and $f^{(64)}(0) = 64!$.

Solution to Problem 2:

Suppose that such a sequence exists. Dividing by $x_{i-1}x_ix_{i+1}$,

$$\frac{2}{x_i} > \frac{1}{x_{i+1}} + \frac{1}{x_{i-1}}$$

$$\frac{1}{x_{i+1}} - \frac{1}{x_i} < \frac{1}{x_i} - \frac{1}{x_{i-1}}$$

Let $p_i = \frac{1}{x_i} - \frac{1}{x_{i-1}}$; we see that p_i is a decreasing sequence. Choose j so that p_j is nonzero. (If this cannot be done, then x_i is constant, which is clearly impossible because of the strict inequality.)

Case 1: $p_j < 0$. The telescoping sum

$$\frac{1}{x_j} + p_{j+1} + p_{j+2} + ... + p_{j+i}$$

is equal to $\frac{1}{x_{i+j}}$. Let k be an integer greater than $\frac{-1}{x_j p_j}$; then

$$\begin{aligned} \frac{1}{x_{j+k}} &= \frac{1}{x_j} + p_{j+1} + p_{j+2} + ... + p_{j+k} \\ &< \frac{1}{x_j} + k \cdot p_j \end{aligned}$$

because p_i is decreasing, and $k > \frac{-1}{x_j p_j}$ implies $k * p_j < \frac{-1}{x_j}$ because p_j is negative. Thus,

$$\frac{1}{x_{j+k}} < \frac{1}{x_j} + k \cdot p_j < \frac{1}{x_j} - 1 - x_j = 0$$

a contradiction.

Case 2: $p_j > 0$. Reverse the order of the sequence (the condition of the problem statement does not change) and p_j becomes negative. Thus a contradiction follows as in case 1.

New Problems

Problem 1: *An End to Charity*

In a group of n people, each person possesses an initial amount of money measured in whole dollars. The group makes a series of two-person transfers under the requirements that: (1) transfers are made only in whole dollars, and (2) each transfer must leave the giver with more money than the taker. Show that only a finite number of such transfers can be made.

Problem 2: *Slippery Subsets*

S is a set of 2000 elements, and P is a set of subsets of S such that if A and B are in P, A is not a proper subset of B. Find the maximum number of elements that P can contain.

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