

## DUKE MATH MEET 2009: RELAY ROUND

In the Relay Round each team of six students will divide into two groups of three students each. There are two sub-rounds in the Relay Round, each group of three students will work together. In each of the two sub-rounds, the three students in a group will receive a different problem. The problems that the second and third students in each group will have the symbol TNYWR within the problem statement. This stands for The Number You Will Receive. This is because the first student in the group is supposed to write down the answer to his or her problem and pass that answer to the second student, and similarly the second student is to pass the answer to his or her problem to the third students. The second student will need the first students answer to completely solve the second question, and similarly the third student will need the second students answer to completely solve the third question. The first and second students can only pass numbers that are fully simplified to the second and third students; there are to be no stray symbols, algebraic expressions, or other marks. The one exception is that students can underline numbers to indicate, for example, whether their answer is a 6 or a 9.

The first and second students may pass as many answers as they wish; however, the third student can only submit answers after three minutes or six minutes, and only the third student's answer will be graded. If a group submits an answer after six minutes then their answer after three minutes, if they gave one, will be discarded. If a group obtains the correct answer after three minutes, they will earn 4 points for their team. If a group obtains the correct answer after six minutes, they will earn 2 points for their team. Therefore, a group should not, for example, submit the same answer after six minutes that they did after three minutes. The moderator will give 15-second warnings before the three minute deadline and the six minute deadline.

### FIRST RELAY ROUND

- 1A. Find the lowest positive angle  $\theta$  that satisfies the equation  $\sqrt{1 + \cos \theta} = \sin \theta + \cos \theta$ , expressed in degrees.
- 1B. Let  $n$  be two times the tens digit of TNYWR. Find the coefficient of the  $x^{n-1}y^{n+1}$  term in the expansion of  $(2x + \frac{y}{2} + 3)^{2n}$ .
- 1C. Let  $k$  be TNYWR, and let  $n = k/2$ . Find the smallest integer  $m$  greater than  $n$  such that 15 divides  $m$  and 12 divides the number of positive integer factors of  $m$ .

### SECOND RELAY ROUND

- 2A. Six werewolves are trapped in a regular hexagonal pen where all side lengths are 2. Each werewolf is tied to a different corner with a leash of length 1. What is the area in the pen that no werewolf can reach?
- 2B. Let  $k$  be TNYWR and let  $m = (k + 2\pi)/\sqrt{3}$ . You have a fair die with  $m$  faces, numbered 1 through  $m$ . You continue to roll that die until you get a 1. Let  $n$  be the total number of times you roll the die (including the last). What is the probability that  $n$  is odd? Report your answer as a fraction in lowest terms.
- 2C. Let  $k$  be the numerator of TNYWR, and let  $a, b$  be real numbers such that  $a + b = 1$  and  $a^2 + b^2 = k$ . Find  $(a^3 + b^3)/(a^4 + b^4)$ .