Displacement Models for THUNDER Actuators having General Loads and Boundary Conditions

Brian Ball, Ralph C. Smith and Zoubeida Ounaies
CRSC and Department of Mathematics, NC State University.

This poster presents two models for the displacements generated in THUNDER actuators in response to applied voltages for a variety of boundary conditions and exogenous loads. The PDE models which quantify displacements in the actuator due to voltage inputs to the piezoceramic patch are developed from Newtonian principles. Although the PZT dynamics are non-linear, drive levels are assumed to be in a regime where linear piezoelectric relations can be employed. A mixed Galerkin finite element method is developed to discretize the models and their performance is illustrated through comparison with experimental data.

Fluid Dynamics and Image Inpainting

M. Bertalmio, *A. Bertozzi*, and G. Sapiro
Department of Mathematics, Duke University.

Well-posedness of the BBM-equation with Periodic Initial Data

Jerry Bona and *Hongqiu Chen*
Department of Mathematical Sciences, The University of Memphis, Tennessee.

The BBM-equation or regularized long-wave equation $u_t + u_x + uu_x - u_{xxt} = 0$ was originally proposed as an alternative to the KdV-equation. Its well-posedness in $H^1(R)$ and $L_2(R)$ have been studied by many authors. In this work, it is shown that if the initial data $u(x, 0)$ is periodic and square integrable within one period interval, then the problem is globally well-posed in time t.

On the Stability of a Class of Self-Similar Solutions to the Filtration-Absorption Equation

Alina Chertock
Lawrence Berkeley National Laboratory.

The equation

$$u_t = u\Delta u - (c-1)(\nabla u)^2 \ , \quad c > 1,$$

is known as a qualitative model of some biological phenomena, but is also a model of groundwater flow in water-absorbing fissurized porous rock. My collaborators and I have considered the one-dimensional case and introduced a family of self-similar solutions with shrinking support. We have investigated numerically the evolution of non-self-similar solutions for Cauchy problems. Numerical experiments demonstrate that the self-similar solutions represent intermediate asymptotics for the solutions to the Cauchy problems with arbitrary initial data of compact support.
As a follow-on, I have constructed a family of axisymmetric self-similar solutions with shrinking support in two dimensions. I have addressed issues of uniqueness and stability for the self-similar solutions, and proved the linear stability of these solutions both in the one- and two-dimensional case. I have also shown that self-similar solutions of the filtration-absorption equation are not uniquely determined, and constructed self-similar solutions which are different from the above solutions. Numerical experiments indicate, however, that the additional self-similar solutions are structurally unstable. Much work is in progress in order to clarify the last point and to provide a definite answer to the nonuniqueness question. Examples from population biology and population genetics are discussed.

Granular Flow in an Annulus

Richard Clelland and Will Kirwin
Dept. of Mathematics, University of Colorado, Boulder.

We present a model for a granular material and use it to numerically simulate flow in an annular region. The model includes a non-associate flow rule and may change type which we associate with the formation of a shear band.

Painleve Singularity Structure Analysis of Landau-Lifshitz Equation in the Vector Form

Louis Kavitha and M. Daniel
Department of Physics, Lady Doak College, INDIA.

Though Painleve singularity structure analysis is an algorithmic procedure to test whether the given system of nonlinear partial differential equations is free from movable critical manifolds and hence expected to be integrable, in some special cases the test is not a straight forward exercise as expected. Heisenberg spin equation, otherwise known as Landau-Lifshitz equation is an example for this. For instance in the case of the Landau-Lifshitz equation that governs the spin dynamics of one dimensional anisotropic Heisenberg spin chain in the presence of a transverse magnetic field, the solution should be assumed in the form of a Taylor series instead of Laurent series while carrying out Painleve analysis in an equivalent representation. It is more of interest to carry out Painleve analysis on the vector form of the equation itself because it was found to be tedious and cumbersome. In this case the length constraint for the spins which has to be checked at each order during the Painleve analysis is also of importance. For the first time we have carried out the Painleve singular point analysis on the vector form of the coupled Landau-Lifshitz equation(in the isotropic case). The above coupled Landau-Lifshitz equation models the spin dynamics of an isotropic spin ladder and from the results of the Painleve analysis we conclude that the system passes the Painleve test when the two legs are locked together at least in the leading order.

An Energy-based Model for Hysteresis in Shape Memory Alloys

Jordan E. Massad, Ralph C. Smith
Department of Mathematics, NC State University.

We develop a model that quantifies constitutive nonlinearities and hysteresis inherent to ferroelastic compounds, with emphasis placed on shape memory alloys. We formulate the model in two steps. First, we use the Landau theory of phase transitions to characterize the effective Gibbs free energy for ferroelastics. The resulting nonlinear equations model ideal material behavior in the absence of impurities. Second, we incorporate pinning losses to account for the energy required to move domain walls across material inclusions. We illustrate aspects of the model through comparison with experimental stress-strain data.

Bell's Inequality and Nonlinear Dynamics

Wm. C. McHarris
Department of Chemistry, Michigan State University.

In the most straightforward interpretation, Bell's inequality should be obeyed by classical dynamics but violated by quantum mechanics. A number of experiments measuring the polarization of entangled photon pairs have demonstrated its violation, and this has been interpreted as ruling out the existence of local interactions in the so-called hidden variables extensions of quantum mechanics. However, Bell's inequality was derived basically using linearly-based statistics, and one can obtain nonlinear formulations of it more or less consistent with the experimental outcomes. This raises the question as to whether the violation of Bell's inequality is moot for ruling out local hidden variables. Indeed, this problem is but one of the "imponderables" generated by quantum mechanics that are at least consistent with some possible chaotic underpinnings for quantum mechanics. [Wm. C. McHarris, Z. f. Naturforsch. 56a, 208 (2001)].

Whip Waves

Tyler McMillen and Alain Goriely
University of Arizona Program in Applied Mathematics.

The crack of a whip is a shock wave created by the supersonic motion of the tip of the whip in air. The whip is modeled as an elastic rod, and a simple two-dimensional model is presented to model the propagation of waves in the tapered rod. The effects of tapering, the boundary conditions, and tension at the handle end are studied theoretically and numerically.

A Two-Current Model for the Dynamics of Cardiac Membrane

Colleen C. Mitchell and David G. Schaeffer
Department of Mathematics, Duke University.

In this paper we introduce and study a model for electrical activity in the heart that incorporates only an inward and an outward current. This model is useful for three reasons: (1) Its complexity is comparable to the FitzHugh-Nagumo model making it useful in numerical simulations, especially in two or three spatial dimensions where numerical efficiency is so important. (2) It can be understood analytically without recourse to numerical simulations. This allows us to determine rather completely how the various parameters in the model affect its behavior. (3) It naturally gives rise to a one- dimensional map which gives action potential duration as a function of the previous diastolic interval. This map has a richer variety of behavior than the commonly used exponential map. For all reasonable parameter values, this model has a region of bistability under periodic pacing. We divide the parameter space into regions with 2:2 - 2:1 bistability, 1:1 - 2:1 bistability, or also give ranges of the key parameters which result in the qualitatively distinct behaviors of stable alternans, unstable alternans, and no Because it can be understood analytically, this model provides insight into the effects of the many parameters in more realistic models.

Robust Control of High Performance Nonlinear Smart Systems

James Nealis and Ralph C. Smith
Department of Mathematics NC State University.

The increasing employment of smart structures in industrial processes necessitates the study of the constitutive nonlinearities and hysteresis exhibited by these materials. Transducers utilizing piezoceramic, electrostrictive, or magnetostrictive elements can often achieve the high performance demands of such processes. These smart structures provide several benefits such as the ability to generate large strains, high bandwidths, or precision placement. However, to utilize these structures to their full potential, models and control laws which accommodate the inherent nonlinearities and hysteresis must be employed. A robust control approach proves to be effective in attenuating the nonlinearities and hysteretic effects of the transducer to achieve the high performance demands of many industrial applications. We illustrate here the performance of a robust control design for a magnetostrictive transducer and note modifications required to extend the theory to systems utilizing piezoceramic or electrostrictive elements.

Multi-Scale Continuum Mechanics: From Global Bifurcations to Noise Induced High Dimensional Chaos

Ira B. Schwartz, David Morgan, Ioana Triandaf, Lora Billings
US Naval Research Laboratory.

Many mechanical systems consist of continuum mechanical structures, having either linear or nonlinear elasticity or geometry, coupled to nonlinear oscillators. In this paper, we consider the class of linear continua coupled to mechanical pendula. In such mechanical systems, there often exists several natural time scales determined by the physics of the problem. Using a time scale splitting, we analyze a prototypical structural/mechanical system consisting of a planar nonlinear pendulum coupled to a flexible rod made of linear viscoelastic material. In this system both low-dimensional and high-dimensional chaos is observed. The low-dimensional chaos appears in the limit of small coupling between the continua and oscillator, where the natural frequency of the primary mode of the rod is much greater that the natural frequency of the pendulum. In this case, the motion resides on a slow manifold. As the coupling is increased, global motion moves off of the slow manifold and high-dimensional chaos is observed. We present a numerical bifurcation analysis of the resulting system illustrating the mechanism for the onset of high dimensional chaos. Practical implications of the bifurcation from low dimensional to high dimensional chaos for detection of damage as well as global effects of noise on a low order model will also be presented.

Attractors for Discrete Periodic Dynamical Systems and Applications to Population Models

James F. Selgrade and John E. Franke
Department of Mathematics, North Carolina State University.

This work presents a simple framework for studying attractors of discrete, nonautonomous dynamical systems which depend periodically on time. Such a nonautonomous dynamical system determines an autonomous system on a topological cylinder in a natural way. The standard definitions for concepts such as invariant set and attractor may be used to define these notions in the time-dependent setting. It is shown that an attractor for a dynamical system of period p is the union of p subsets which, under certain conditions, are homeomorphic to one another. In addition, if the periodic system is a perturbation of an autonomous system then conditions are presented which guarantee that each subset is homeomorphic to an attractor for the autonomous system.

Granular jets

Amy Shen, Siggi Thoroddsen
Division of Engineering and Applied Sciences, Harvard University.

When a solid sphere impacts on a deep layer of granular medium, it generates an ejecta sheet and a transient axisymmetric crater. The gravity-driven radial collapse of this crater generates a pressure spike as the cavity closes. This pressure spike drives up a narrow granular jet along the axis of symmetry.
The maximum height of the granular jet is found to depend on the impact velocity, gravity, and the effective viscosity of the granular medium through a simple product of the Reynolds and Froude numbers. The presence of such granular jets, where surface tension is absent, may help pinpoint the role of surface tension for similar liquid jets. This experiment may also give insights into the constitutive properties of flowing granular media at high shear-rates.

The Axisymmetric Stretching of a Non-Newtonian Liquid Filament and a New Exact Solution

Linda B. Smolka, A. Belmonte and D. M. Henderson
Department of Mathematics, Duke University.

We generalize the free boundary problem for an infinitely-long cylindrical liquid filament stretching in a purely extensional flow and derive a general condition for the existence of solutions. This approach provides a systematic framework which allows various Newtonian and non-Newtonian constitutive models to be evaluated. For the Upper Convected Maxwell model (a viscoelastic model) we have found an analytic solution for the filament motion. We compare this solution with experimental measurements of the thinning of dilute polymer solutions. Our equation captures one aspect of the motion: that the filament thickness initially decreases exponentially, then slows at larger times, scaling in the asymptotic limit $t \to \infty$ as $1/\sqrt{t}$, which is the scaling of the known Newtonian exact solution. This change provides insight as to how the molecular dynamics couple to the filament's motion.

The Quantum Interaction of Charged Particles and Monochromatic Radiation in a Medium

Armen Stepanyan
Yerevan State University, Republic of Armenia.

We study the quantum theory of nonlinear interaction of charged particles and a given field of plane-electromagnetic radiation in a medium. The approximate nonlinear solution of the Mathieu equation to which the relativistic quantum equation of particle motion in the given field reduces if one ignores the spin-field interaction is found (the Klein-Gordon equation). At the high energy particle motion the obtained solution is valid in the case if the energy change of particle is in the order of particle energy in contrast to the Eikonal approximation. At the small field interaction the obtained solution is the analytic expression of the infinite sum of perturbation theory in the field. We study the stability of solutions and find a class of restricted solutions corresponding to the wave function of the particle. The method developed in the paper can be applied to a broad class of problems reducible to the solution of the Mathieu equation.

Pinch-off transition of thin liquid films driven by Marangoni forces and gravity
Jeanman Sur, Robert P. Behringer and Andrea L. Bertozzi
Department of Physics, Duke University.

Thin liquid films pinch off from the bulk under the influence of competing effects of Marangoni force and gravity, when a substrate is dipped in a liquid reservoir and then pulled out. The pinch-off transition first observed by Ludviksson and Lightfoot [AIChE J. 17, 1166(1971)] has been studied theoretically by Andreas Muench [preprint, sumitted to PRL] recently. Muench's results show that the pinch-off transition gives rise to a leading undercompressive shock, followed by a new reverse undercompressive shock and trailing rarefraction wave. We reproduce his results and compare theoretical results with experiments. We consider the possibility of pinchoff dynamics involving three shocks: two undercompressive shocks and a third compressive wave.

Subharmonics for first order convex nonautonomous Hamiltonian systems
Xiangjin Xu
Department of Mathematics, Johns Hopkins University.

In this paper new estimates on the C^0-norm of solutions are shown for first order convex Hamiltonian systems possessing super-quadratic potentials. Applying these estimates, some new results on the subharmonics are gained, which generalize the main results in [Ekeland, I. and Hofer, H. Subharmonics for convex non-autonomous Hamiltonian systems. Comm. PAM., 40 (1987) 1 – 36], and a question about *a priori* estimates on subharmonics raised by Ekeland and Hofer is answered when the convex Hamiltonian systems have globally super-quadratic potentials.

The weak shear problem for nematic polymers
M. G. Forest, Q. Wang, and *R. Zhou*
Applied Mathematics, University of North Carolina-Chapel Hill.

We study the weak shear problem of nematic polymers modeled by the kinetic theory of Doi. The probability distribution function (pdf) is to be solved for the orientation of rigid molecules due to the interaction of an excluded-volume potential and an imposed shear flow. This problem is the classical unsolved dynamics problem in nematic liquids, at least from a rigorous mathematical perspective. The heart of the problem is that *prior to flow* nematic polymers are *orientationally degenerate*: the Smoluchowski equation has an invariant O(3) symmetry group. Weak flows such as shear break this symmetry, and the stationary distributions which survive are unknown with respect to number, type, and stability. We report theoretical and numerical progress on this problem.

On Exponentially Small Effects in Dynamical Systems with Small Parameters
Oleg Zubelevich
Institute of Theoretical Engineering, Moscow, Russia.

Theorem about exponentially small effects is considered. This theorem enables to obtain exponentially small effects in different dynamical systems with small parameter.
